

# MECHANIZED SEMANTICS AND VERIFIED COMPILATION FOR A DATAFLOW SYNCHRONOUS LANGUAGE WITH RESET

PHD THESIS DEFENSE

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## CONTEXT

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## Embedded systems

- computer systems within physical systems that interact with the real world, often with real-time constraints
- software usually written in low-level languages: C, Ada, Assembly



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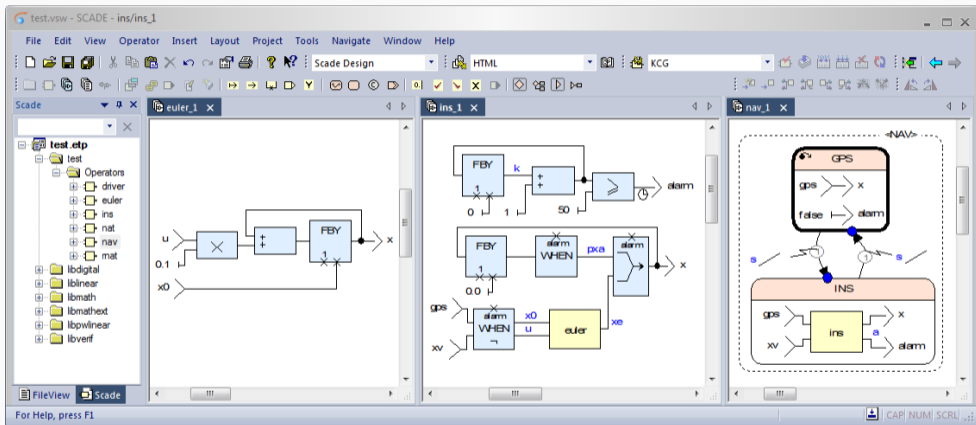
### Model-Based Design

Executable high-level abstract specifications



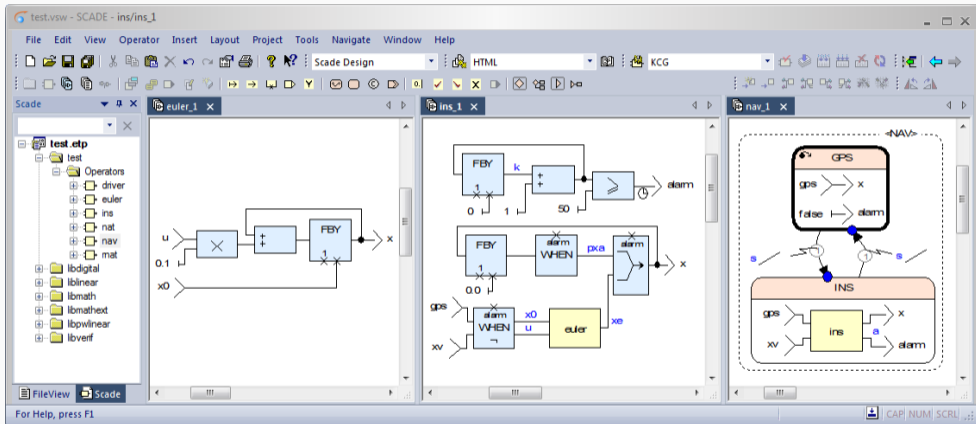
## MODEL-BASED DESIGN IN SCADE SUITE

[www.ansys.com/products/embedded-software/ansys-scade-suite](http://www.ansys.com/products/embedded-software/ansys-scade-suite)



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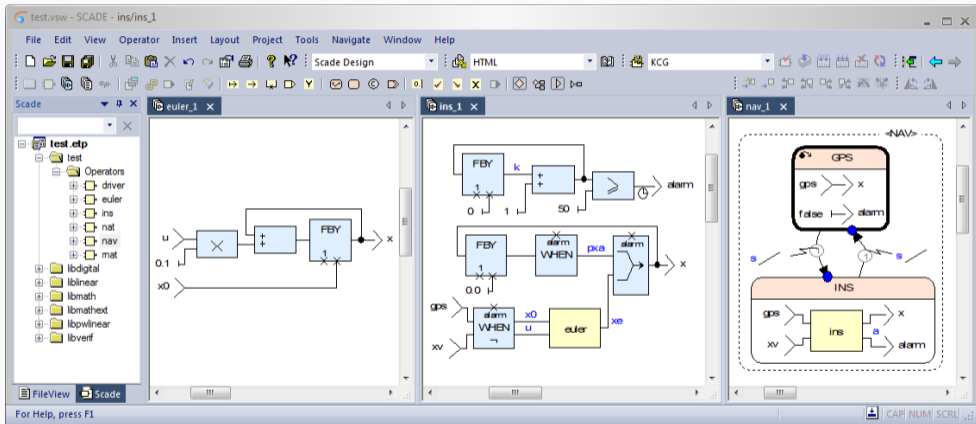


block / node = system

line = signal

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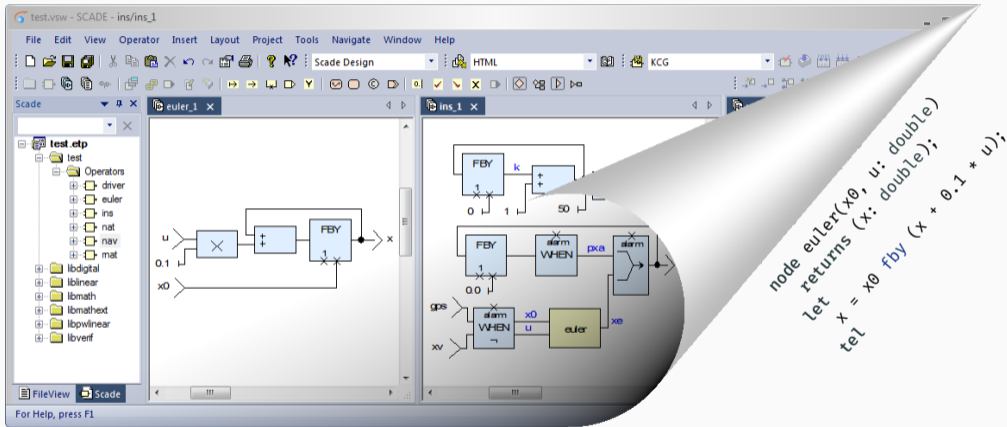
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block / node = system = stream function  
 line = signal = stream of values

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# MODEL-BASED DESIGN IN SCADE SUITE

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The screenshot shows the SCADE Suite interface with two windows. The left window, titled 'euler\_1', displays a block diagram with inputs 'u' and 'x0', a multiplier block '×', an adder block '+', and an 'FBY' (Feedback) block. The right window, titled 'ins\_1', shows a more complex block diagram with 'FBY' blocks, 'alarm WHEN' blocks, and an 'euler' block. A callout bubble points from the 'FBY' block in the right window to a code snippet:

```
node euler(x0, u: double)
returns (x: double);
let
  x = x0 fby (x + 0.1 * u);
tel
```

An arrow points from the code snippet to the text 'sequential program (C, Ada, assembly)'.

block / node = system = stream function  
 line = signal = stream of values

## Systems that must not fail

- Flight control systems
- Automated train systems
- Power plant monitoring software



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**State of the art:** **industrial certification** of the development process, sometimes using *formal methods*, eg. SCADE

**Scientific question:** can we **mechanize** the formal definitions and produce an **end-to-end correctness proof**?

## Interactive Theorem Provers, or Proof Assistants

- Software tools to assist statement of theorems, and development and checking of their proofs
- Mizar, Isabelle, HOL, Coq, ACL2, PVS, Agda, ...



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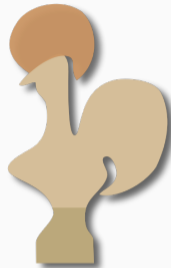
## Existing mechanized formalizations

**seL4**: a verified micro-kernel in Isabelle

**CakeML**: a verified compiler for a functional language in HOL

### **CompCert: a milestone**

Formal mechanization in Coq of the C language and of the correctness proof of its compilation to Assembly code.



## Model-Based Design Languages

Scade 6, Lustre



## Interactive Theorem Provers

Coq

### Challenges

1. Mechanize the semantics
2. Prove the compilation algorithms correct

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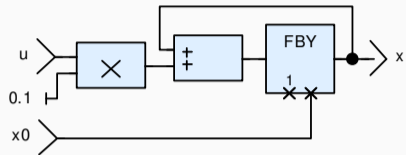
Coq

### Challenges

1. Mechanize the semantics
2. Prove the compilation algorithms correct

Focus: modular reset

## EXAMPLE

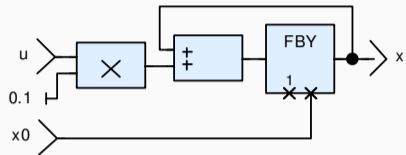


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$x_0$	0.00	1.55	3.62	5.46	...
$u$	15.00	20.00	17.00	12.00	...
<hr/>					
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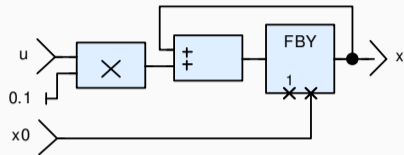
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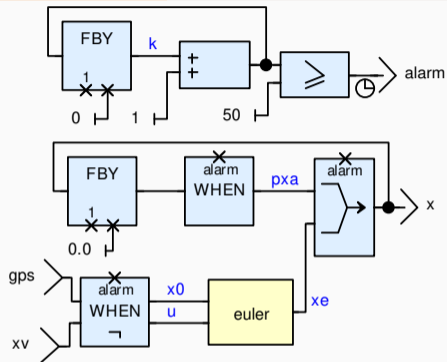
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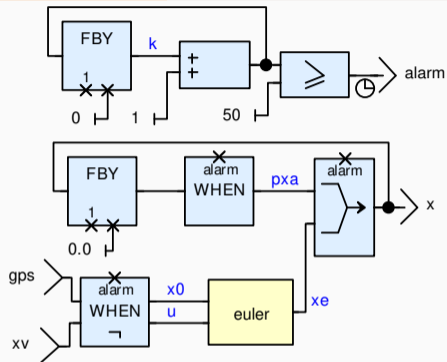
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  returns (x: double, alarm: bool)
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  k = 0 fby (k + 1);
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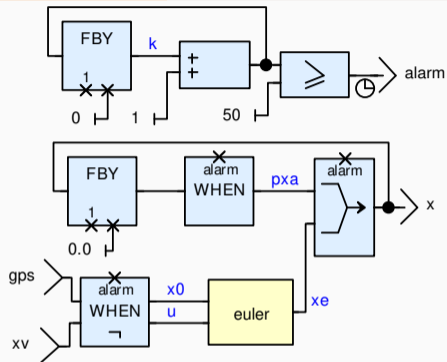
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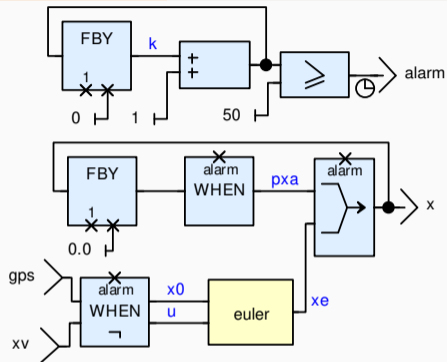
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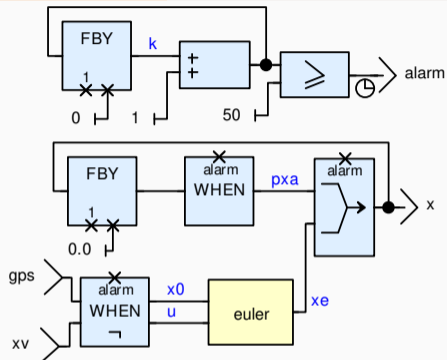
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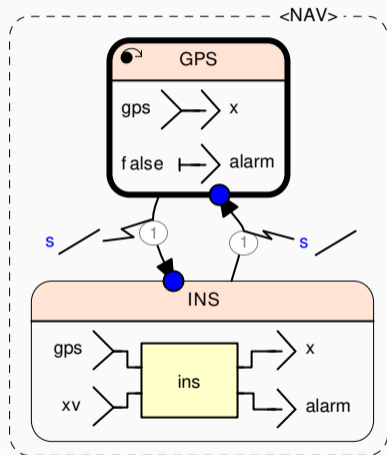
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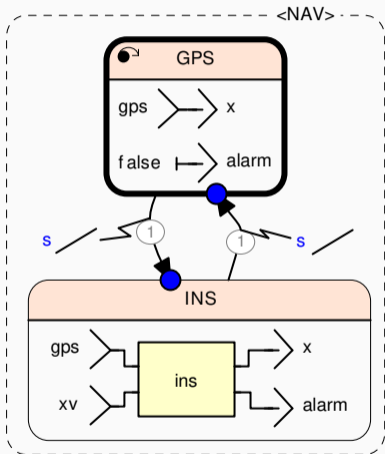
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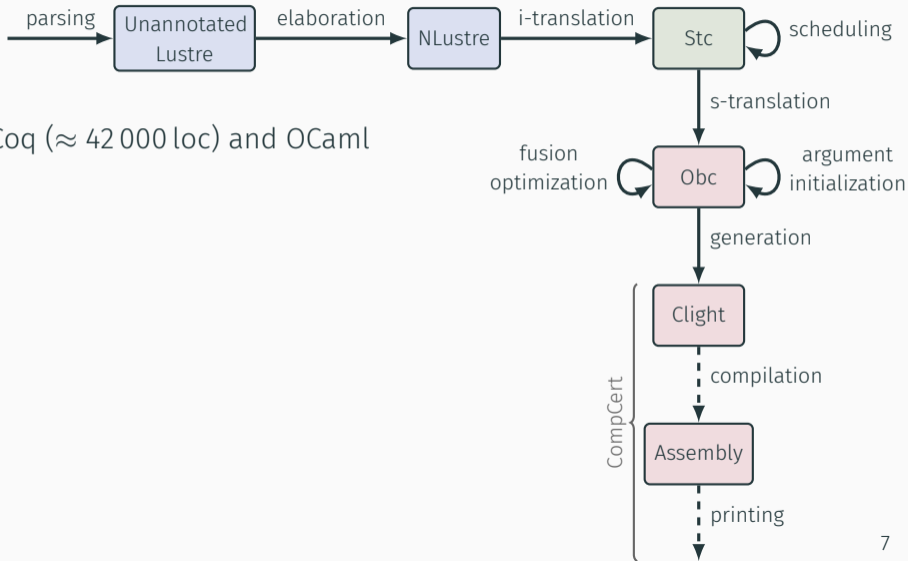
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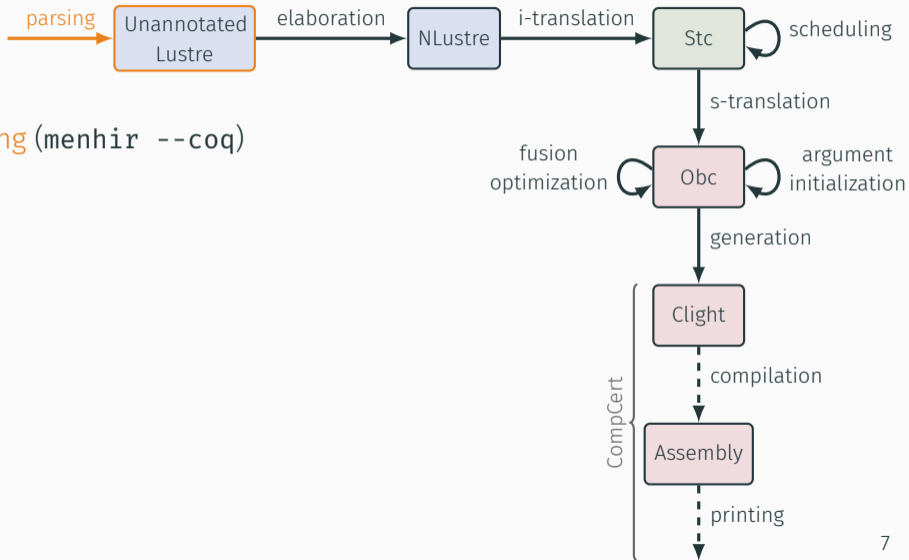
```

We need a way to **reset the state of a node**

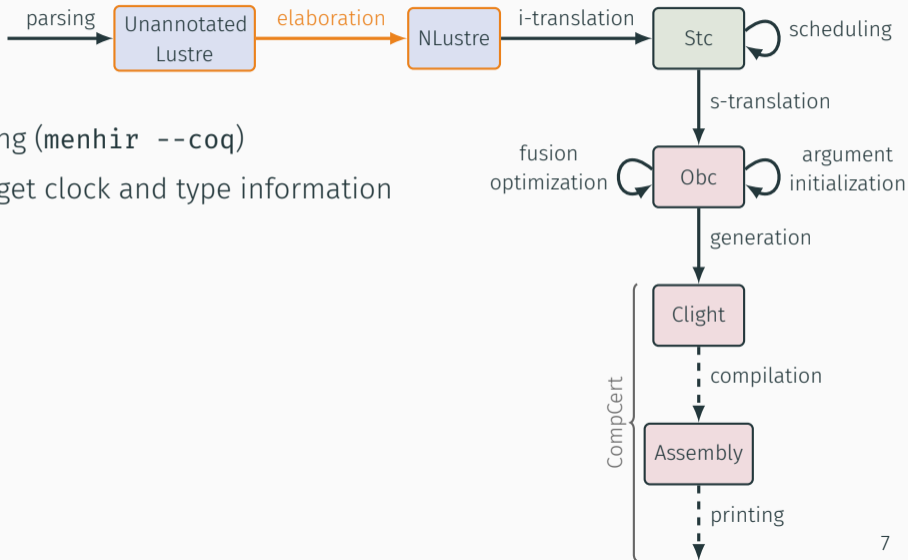
Implemented in Coq ( $\approx 42\,000$  loc) and OCaml



- validated `parsing` (`menhir --coq`)

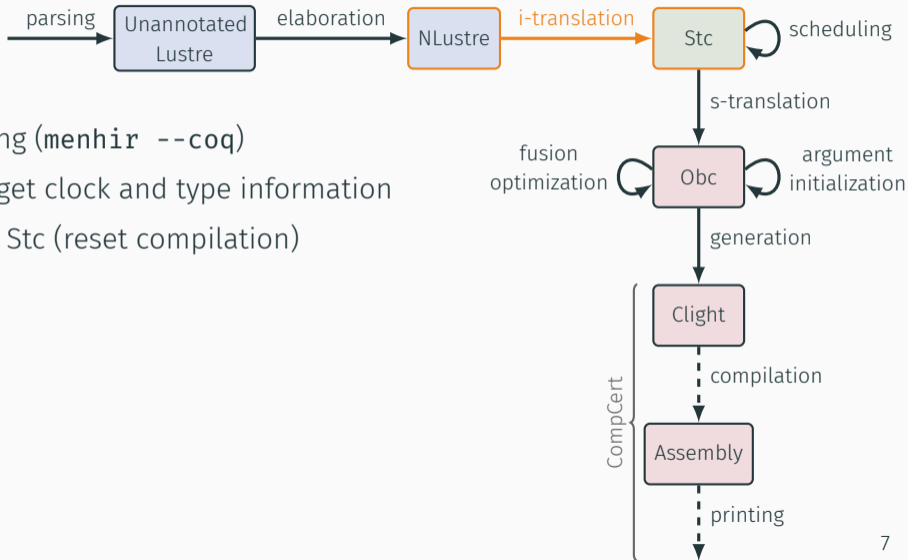


# VÉLUS: A VERIFIED LUSTRE COMPILER



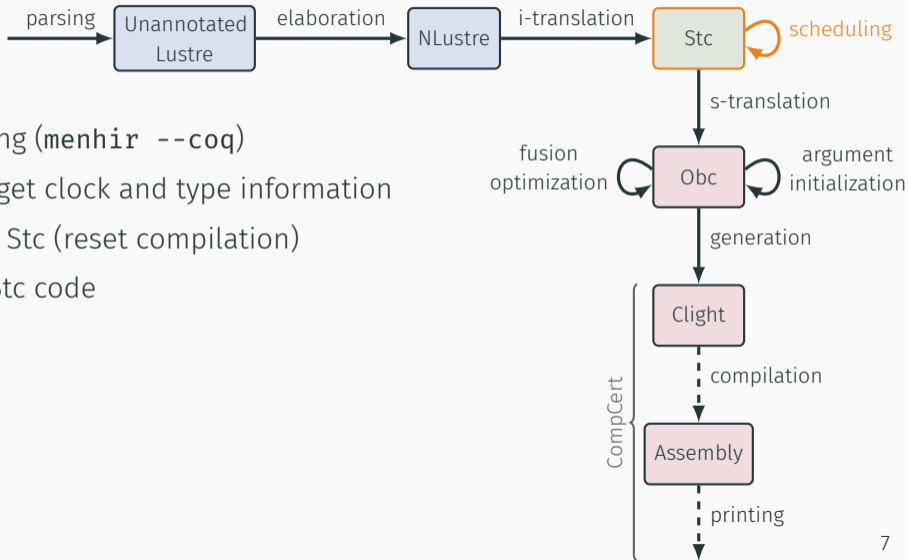
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- **elaboration** to get clock and type information

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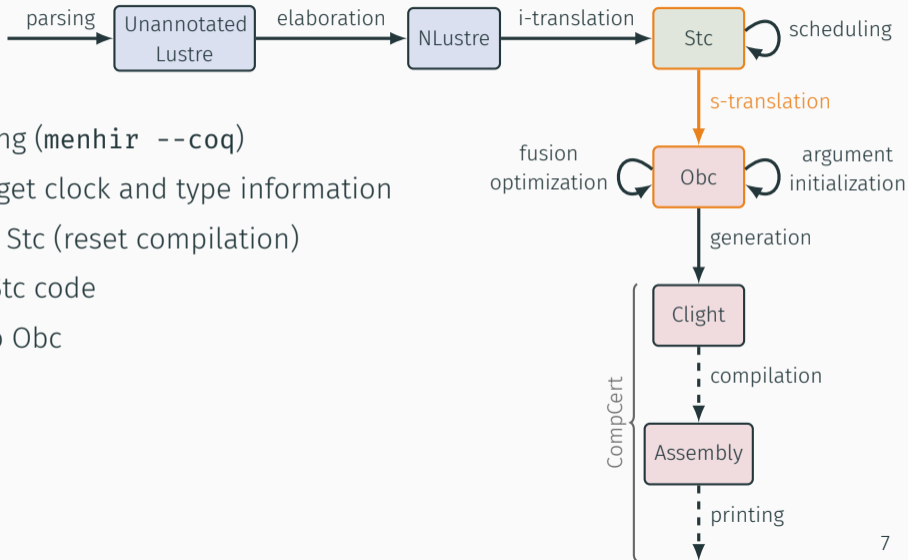
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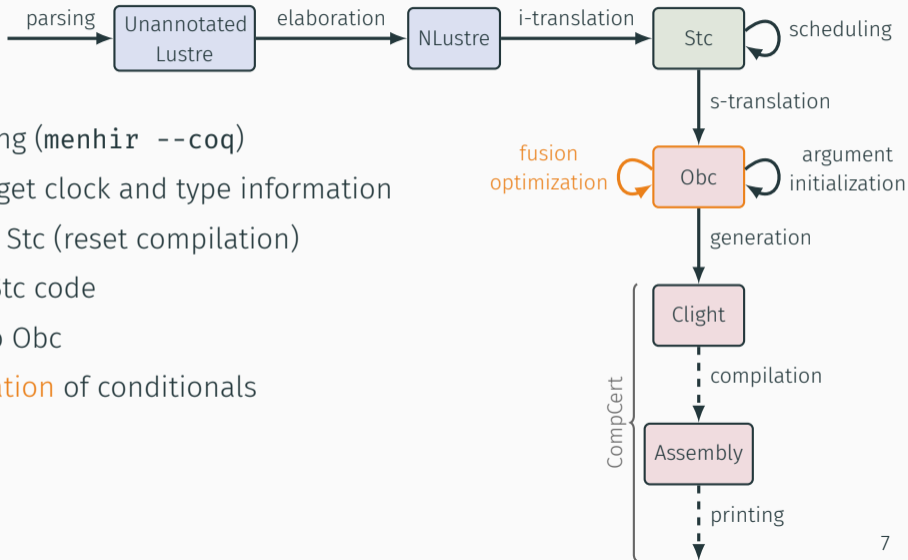
- validated parsing (`menhir --coq`)
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- **scheduling** of Stc code

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- **s-translation** to Obc

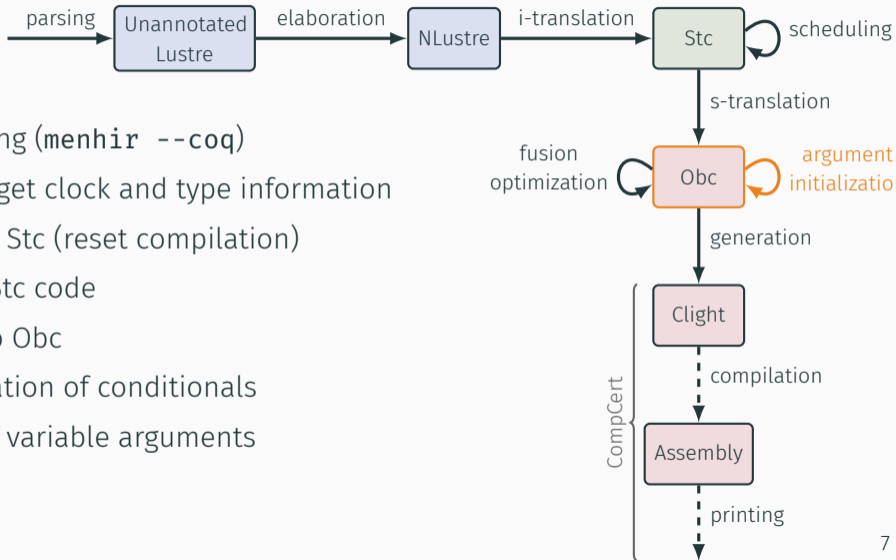
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- **fusion optimization** of conditionals

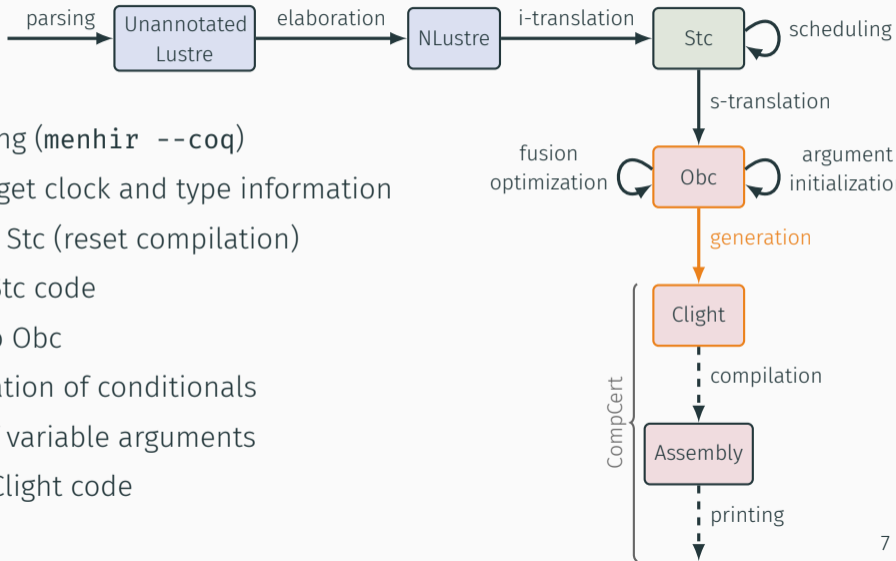


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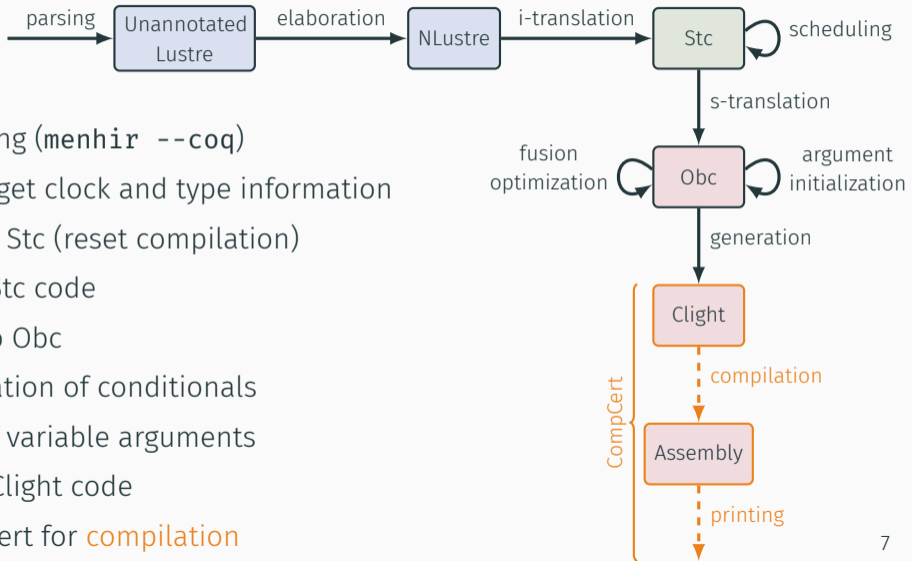
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- **Generation** of Clight code

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- Generation of Clight code
- Rely on CompCert for **compilation**

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node euler(x0, u: double)
  returns (x: double);
let
  x = x0 fby (x + 0.1 * u);
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node ins(gps, xv: double)
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```

```

struct euler {
  bool i;
  double gx;
};
struct ins {
  int k;
  double gx;
  struct euler xe;
};
struct fun$ins$step {
  double x;
  bool alarm;
};
struct nav {
  bool c;
  bool r;
  struct ins insr;
};
struct fun$nav$step {
  double x;
  bool alarm;
};

double fun$euler$step(struct euler *self,
                     double x0, double u) {
  register double x;
  if (self->i) {
    x = x0;
  } else {
    x = self->gx;
  }
  self->i = false;
  self->gx = x + 0.10000000000000000 * u;
  return x;
}

void fun$euler$reset(struct euler *self) {
  self->i = true;
  self->gx = 0;
  return;
}

void fun$ins$step(struct ins *self,
                  struct fun$ins$step *out,
                  double gps, double xv) {
  register double step$;
  register double xe;
  out->alarm = self->k >= 50;
  self->k = self->k + 1;
  if (out->alarm) { out->x = self->gx; }
  else {
    step$ = fun$euler$step(&self->xe, gps, xv);
    xe = step$;
    out->x = xe;
  }
  self->gx = out->x;
  return;
}

void fun$ins$reset(struct ins *self) {
  self->k = 0;
  self->gx = 0;
  fun$euler$reset(&self->xe);
  return;
}

void fun$nav$step(struct nav *self,
                  struct fun$nav$step *out,
                  double gps, double xv, bool s) {
  struct fun$ins$step out$insr$step;
  register bool cm;
  register double insr;
  register bool alr;
  if (self->r) { fun$ins$reset(&self->insr); }
  self->r = s & self->c;
  if (self->c) {
    cm = !s;
    out->x = gps;
    out->alarm = false;
  } else {
    fun$ins$step(&self->insr, &out$insr$step, gps, xv);
    insr = out$insr$step.x;
    alr = out$insr$step.alarm;
    cm = s;
    out->x = insr;
    out->alarm = alr;
  }
  self->c = cm;
  return;
}

void fun$nav$reset(struct nav *self) {
  self->c = true;
  self->r = false;
  fun$ins$reset(&self->insr);
  return;
}

struct nav self$;
double volatile gps$;
double volatile xv$;
bool volatile s$;
double volatile x$;
bool volatile alarm$;

int main(void) {
  struct fun$nav$step out$step;
  register double gps;
  register double xv;
  register bool s;

  fun$nav$reset(&self$);

  while (true) {
    gps = volatile_load(&gps$);
    xv = volatile_load(&xv$);
    s = volatile_load(&s$);

    fun$nav$step(&self$, &out$step, gps, xv, s);

    volatile_store(&x$, out$step.x);
    volatile_store(&alarm$, out$step.alarm);
  }
}

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tel

node ins(gps, xv: double)
  returns (x: double, alarm: bool)
  var pxa, xe: double; k: int;
let
  k = 0 fby (k + 1);
  alarm = (k >= 50);
  xe = euler((gps, xv) when not alarm);
  pxa = (0. fby x) when alarm;
  x = merge alarm pxa xe;
tel

node nav(gps: double, xv: double, s: bool)
  returns (x: double, alarm: bool)
  var r: bool, c: bool;
let
  (x, alarm) = merge c
    (gps when c, false)
    ((restart ins every r)
     ((gps, xv) whenot c));
  c = true fby (merge c (not s when c)
                (s whenot c));
  r = false fby (s and c);
tel

```

```

struct euler {
  bool i;
  double gx;
};
struct ins {
  int k;
  double gx;
  struct euler xe;
};
struct fun$ins$step {
  double x;
  bool alarm;
};
struct nav {
  bool c;
  bool r;
  struct ins insr;
};
struct fun$nav$step {
  double x;
  bool alarm;
};

double fun$euler$step(struct euler *self,
                     double x0, double u) {
  register double x;
  if (self->i) {
    x = x0;
  } else {
    x = self->gx;
  }
  self->i = false;
  self->gx = x + 0.10000000000000000 * u;
  return x;
}

void fun$euler$reset(struct euler *self) {
  self->i = true;
  self->gx = 0;
  return;
}

void fun$ins$step(struct ins *self,
                 struct fun$ins$step *out,
                 double gps, double xv) {
  register double step$;
  register double xe;
  out->alarm = self->k >= 50;
  self->k = self->k + 1;
  if (out->alarm) { out->x = self->gx; }
  else {
    step$ = fun$euler$step(&self->xe, gps, xv);
    xe = step$;
    out->x = xe;
  }
  self->gx = out->x;
  return;
}

void fun$ins$reset(struct ins *self) {
  self->k = 0;
  self->gx = 0;
  fun$euler$reset(&self->xe);
  return;
}

void fun$nav$step(struct nav *self,
                 struct fun$nav$step *out,
                 double gps, double xv, bool s) {
  struct fun$ins$step out$insr$step;
  register bool cm;
  register double insr;
  register bool alr;
  if (self->r) { fun$ins$reset(&self->insr); }
  self->r = s & self->c;
  if (self->c) {
    cm = !s;
    out->x = gps;
    out->alarm = false;
  } else {
    fun$ins$step(&self->insr, &out$insr$step, gps, xv);
    insr = out$insr$step.x;
    alr = out$insr$step.alarm;
    cm = s;
    out->x = insr;
    out->alarm = alr;
  }
  self->c = cm;
  return;
}

void fun$nav$reset(struct nav *self) {
  self->c = true;
  self->r = false;
  fun$ins$reset(&self->insr);
  return;
}

struct nav self$;
double volatile gps$;
double volatile xv$;
bool volatile s$;
double volatile x$;
bool volatile alarm$;

int main(void) {
  struct fun$nav$step out$step;
  register double gps;
  register double xv;
  register bool s;

  fun$nav$reset(&self$);

  while (true) {
    gps = volatile_load(&gps$);
    xv = volatile_load(&xv$);
    s = volatile_load(&s$);

    fun$nav$step(&self$, &out$step, gps, xv, s);

    volatile_store(&x$, out$step.x);
    volatile_store(&alarm$, out$step.alarm);
  }
}

```

translated code

```

node euler(x0, u: double)
  returns (x: double);
let
  x = x0 fby (x + 0.1 * u);
tel

node ins(gps, xv: double)
  returns (x: double, alarm: bool)
  var pxa, xe: double; k: int;
let
  k = 0 fby (k + 1);
  alarm = (k >= 50);
  xe = euler((gps, xv) when not alarm);
  pxa = (0. fby x) when alarm;
  x = merge alarm pxa xe;
tel

node nav(gps: double, xv: double, s: bool)
  returns (x: double, alarm: bool)
  var r: bool, c: bool;
let
  (x, alarm) = merge c
    (gps when c, false)
    ((restart ins every r)
     ((gps, xv) whennot c));
  c = true fby (merge c (not s when c)
                (s whennot c));
  r = false fby (s and c);
tel

```

```

struct euler {
  bool i;
  double gx;
};
struct ins {
  int k;
  double gx;
  struct euler xe;
};
struct fun$ins$step {
  double x;
  bool alarm;
};
struct nav {
  bool c;
  bool r;
  struct ins insr;
};
struct fun$nav$step {
  double x;
  bool alarm;
};

double fun$euler$step(struct euler *self,
                     double x0, double u) {
  register double x;
  if (self->i) {
    x = x0;
  } else {
    x = self->gx;
  }
  self->i = false;
  self->gx = x + 0.10000000000000000 * u;
  return x;
}

void fun$euler$reset(struct euler *self) {
  self->i = true;
  self->gx = 0;
  return;
}

void fun$ins$step(struct ins *self,
                 struct fun$ins$step *out,
                 double gps, double xv) {
  register double step$;
  register double xe;
  out->alarm = self->k >= 50;
  self->k = self->k + 1;
  if (out->alarm) { out->x = self->gx; }
  else {
    step$x = fun$euler$step(&self->xe, gps, xv);
    xe = step$x;
    out->x = xe;
  }
  self->gx = out->x;
  return;
}

void fun$ins$reset(struct ins *self) {
  self->k = 0;
  self->gx = 0;
  fun$euler$reset(&self->xe);
  return;
}

```

```

void fun$nav$step(struct nav *self,
                 struct fun$nav$step *out,
                 double gps, double xv, bool s) {
  struct fun$ins$step out$insr$step;
  register bool cm;
  register double insr;
  register bool alr;
  if (self->r) { fun$ins$reset(&self->insr); }
  self->r = s & self->xc;
  if (self->c) {
    cm = !s;
    out->x = gps;
    out->alarm = false;
  } else {
    fun$ins$step(&self->insr, &out$insr$step, gps, xv);
    insr = out$insr$step.x;
    alr = out$insr$step.alarm;
    cm = s;
    out->x = insr;
    out->alarm = alr;
  }
  self->xc = cm;
  return;
}

void fun$nav$reset(struct nav *self) {
  self->c = true;
  self->r = false;
  fun$ins$reset(&self->insr);
  return;
}

struct nav self$;
double volatile gps$;
double volatile xv$;
bool volatile s$;
double volatile x$;
bool volatile alarm$;

int main(void) {
  struct fun$nav$step out$step;
  register double gps;
  register double xv;
  register bool s;

  fun$nav$reset(&self$);

  while (true) {
    gps = volatile_load(&gps$);
    xv = volatile_load(&xv$);
    s = volatile_load(&s$);

    fun$nav$step(&self$, &out$step, gps, xv, s);

    volatile_store(&x$, out$step.x);
    volatile_store(&alarm$, out$step.alarm);
  }
}

```

main loop

# LUSTRE

# ASSEMBLY

```

node euler(x0, u: double)
  returns (x: double);
let
  x = x0 fby (x + 0.1 * u);
tel

node ins(gps, xv: double)
  returns (x: double, alarm: bool)
  var pxa, xe: double; k: int;
let
  k = 0 fby (k + 1);
  alarm = (k >= 50);
  xe = euler((gps, xv) when not alarm);
  pxa = (0. fby x) when alarm;
  x = merge alarm pxa xe;
tel

node nav(gps: double, xv: double, s: bool)
  returns (x: double, alarm: bool)
  var r: bool, c: bool;
let
  (x, alarm) = merge c
    (gps when c, false)
    ((restart ins every r)
     ((gps, xv) whenot c));
  c = true fby (merge c (not s when c)
    (s whenot c));
  r = false fby (s and c);
tel

```

```

node euler(x0, u: double)
  returns (x: double);
let
  x = x0 fby (x + 0.1 * u);
tel

node ins(gps, xv: double)
  returns (x: double, alarm: bool)
  var pxa, xe: double; k: int;
let
  k = 0 fby (k + 1);
  alarm = (k >= 50);
  xe = euler((gps, xv) when not alarm);
  pxa = (0. fby x) when alarm;
  x = merge alarm pxa xe;
tel

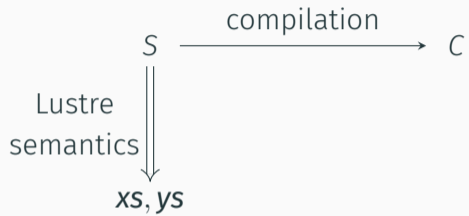
node nav(gps: double, xv: double, s: bool)
  returns (x: double, alarm: bool)
  var r: bool, c: bool;
let
  (x, alarm) = merge c
    (gps when c, false)
    ((restart ins every r)
     ((gps, xv) whenot c));
  c = true fby (merge c (not s when c)
    (s whenot c));
  r = false fby (s and c);
tel

```

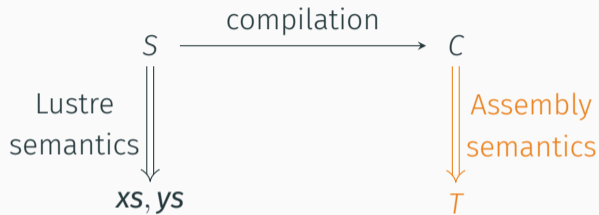


$S \xrightarrow{\text{compilation}} C$

# CORRECTNESS?



# CORRECTNESS?



# CORRECTNESS?



# CORRECTNESS?



**Remark:** we actually want the reverse direction, called *refinement*, that is, the observable behaviors of  $C$  are also observable behaviors of  $S$ .

# NORMALIZED LUSTRE MECHANIZATION

---

# PRESENTATION OF NLUSTRE

4 types of top-level equations

$$x = ce$$

$$x = c \text{ fby } e$$

$$x = f(e)$$

$$x = (\text{restart } f \text{ every } r)(e)$$

simple equation

**fby** equation

node instantiation

node instantiation with modular reset

## Semantics

Streams as functions  $\mathbb{N} \mapsto \text{value}$ :

$$\begin{array}{cccc} 0 & 1 & 2 & \dots \\ \Downarrow & \Downarrow & \Downarrow & \dots \\ v_0 & v_1 & v_2 & \dots \end{array}$$

lifted instantaneous semantics

Streams as coinductive types:

$$v_0 \cdot v_1 \cdot v_2 \cdot \dots$$

coinductive description of the semantics 10

## Instantaneous Semantics

$$\frac{}{R \vdash x \downarrow R(x)} \quad \frac{R \vdash e_1 \downarrow \langle v_1 \rangle \quad R \vdash e_2 \downarrow \langle v_2 \rangle}{R \vdash e_1 + e_2 \downarrow \langle [[+]](v_1, v_2) \rangle} \quad \frac{R \vdash e_1 \downarrow \langle \rangle \quad R \vdash e_2 \downarrow \langle \rangle}{R \vdash e_1 + e_2 \downarrow \langle \rangle}$$

## Lifted Semantics

$$\frac{\forall i, H_i(x) = s_i \quad \forall i, H_i \vdash e \downarrow s_i}{H \vdash x = e} \quad \frac{\forall i, H_i \vdash e \downarrow s_i \quad \forall i, H_i(x) = \text{fby}(c, s)_i}{H \vdash x = c \text{ fby } e}$$

$$\frac{\forall i, H_i \vdash e \downarrow xs_i \quad \vdash f(xs) \Downarrow ys \quad \forall i, H_i(x) = ys_i}{H \vdash x = f(e)}$$



## Instantaneous Semantics

$$\frac{}{R \vdash x \downarrow R(x)} \quad \frac{R \vdash e_1 \downarrow \langle v_1 \rangle \quad R \vdash e_2 \downarrow \langle v_2 \rangle}{R \vdash e_1 + e_2 \downarrow \langle [[+]](v_1, v_2) \rangle} \quad \frac{R \vdash e_1 \downarrow \langle \rangle \quad R \vdash e_2 \downarrow \langle \rangle}{R \vdash e_1 + e_2 \downarrow \langle \rangle}$$

## Lifted Semantics

$$\frac{\forall i, H_i(x) = s_i \quad \forall i, H_i \vdash e \downarrow s_i}{H \vdash x = e} \quad \frac{\forall i, H_i \vdash e \downarrow s_i \quad \forall i, H_i(x) = \text{fby}(c, s)_i}{H \vdash x = c \text{ fby } e}$$

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## Instantaneous Semantics

$$\frac{}{R \vdash x \downarrow R(x)} \quad \frac{R \vdash e_1 \downarrow \langle v_1 \rangle \quad R \vdash e_2 \downarrow \langle v_2 \rangle}{R \vdash e_1 + e_2 \downarrow \langle [[+]](v_1, v_2) \rangle} \quad \frac{R \vdash e_1 \downarrow \langle \rangle \quad R \vdash e_2 \downarrow \langle \rangle}{R \vdash e_1 + e_2 \downarrow \langle \rangle}$$

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$$\frac{\forall i, H_i(x) = s_i \quad \forall i, H_i \vdash e \downarrow s_i}{H \vdash x = e} \quad \frac{\forall i, H_i \vdash e \downarrow s_i \quad \forall i, H_i(x) = \text{fby}(c, s)_i}{H \vdash x = c \text{ fby } e}$$

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## Instantaneous Semantics

$$\frac{}{R \vdash x \downarrow R(x)} \quad \frac{R \vdash e_1 \downarrow \langle v_1 \rangle \quad R \vdash e_2 \downarrow \langle v_2 \rangle}{R \vdash e_1 + e_2 \downarrow \langle [[+]](v_1, v_2) \rangle} \quad \frac{R \vdash e_1 \downarrow \langle \rangle \quad R \vdash e_2 \downarrow \langle \rangle}{R \vdash e_1 + e_2 \downarrow \langle \rangle}$$

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$$\frac{\forall i, H_i(x) = s_i \quad \forall i, H_i \vdash e \downarrow s_i}{H \vdash x = e} \quad \frac{\forall i, H_i \vdash e \downarrow s_i \quad \forall i, H_i(x) = \text{fby}(c, s)_i}{H \vdash x = c \text{ fby } e}$$

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## Instantaneous Semantics

$$\frac{}{R \vdash x \downarrow R(x)} \quad \frac{R \vdash e_1 \downarrow \langle v_1 \rangle \quad R \vdash e_2 \downarrow \langle v_2 \rangle}{R \vdash e_1 + e_2 \downarrow \langle [[+]](v_1, v_2) \rangle} \quad \frac{R \vdash e_1 \downarrow \langle \rangle \quad R \vdash e_2 \downarrow \langle \rangle}{R \vdash e_1 + e_2 \downarrow \langle \rangle}$$

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## Lifted Semantics

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## Instantaneous Semantics

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$$\frac{\forall i, H_i \vdash e \downarrow xs_i \quad \vdash f(xs) \Downarrow ys \quad \forall i, H_i(x) = ys_i}{H \vdash x = f(e)}$$



## Instantaneous Semantics

$$\frac{}{R \vdash x \downarrow R(x)} \quad \frac{R \vdash e_1 \downarrow \langle v_1 \rangle \quad R \vdash e_2 \downarrow \langle v_2 \rangle}{R \vdash e_1 + e_2 \downarrow \langle \llbracket + \rrbracket(v_1, v_2) \rangle} \quad \frac{R \vdash e_1 \downarrow \langle \rangle \quad R \vdash e_2 \downarrow \langle \rangle}{R \vdash e_1 + e_2 \downarrow \langle \rangle}$$

## Lifted Semantics

$$\frac{\forall i, H_i(x) = s_i \quad \forall i, H_i \vdash e \downarrow s_i}{H \vdash x = e} \quad \frac{\forall i, H_i \vdash e \downarrow s_i \quad \forall i, H_i(x) = \text{fby}(c, s)_i}{H \vdash x = c \text{ fby } e}$$

$$\frac{\forall i, H_i \vdash e \downarrow xs_i \quad \vdash f(xs) \Downarrow ys \quad \forall i, H_i(x) = ys_i}{H \vdash x = f(e)}$$

## Instantaneous Semantics

$$\frac{}{R \vdash x \downarrow R(x)} \quad \frac{R \vdash e_1 \downarrow \langle v_1 \rangle \quad R \vdash e_2 \downarrow \langle v_2 \rangle}{R \vdash e_1 + e_2 \downarrow \langle \llbracket + \rrbracket(v_1, v_2) \rangle} \quad \frac{R \vdash e_1 \downarrow \langle \rangle \quad R \vdash e_2 \downarrow \langle \rangle}{R \vdash e_1 + e_2 \downarrow \langle \rangle}$$

## Lifted Semantics

$$\frac{\forall i, H_i(x) = s_i \quad \forall i, H_i \vdash e \downarrow s_i}{H \vdash x = e} \quad \frac{\forall i, H_i \vdash e \downarrow s_i \quad \forall i, H_i(x) = \text{fby}(c, s)_i}{H \vdash x = c \text{ fby } e}$$

$$\frac{\forall i, H_i \vdash e \downarrow xs_i \quad \vdash f(xs) \Downarrow ys \quad \forall i, H_i(x) = ys_i}{H \vdash x = f(e)}$$

## Node Semantics

$$\frac{\text{node}(G, f) = n \quad \forall i, H_i(n.\text{in}) = xs_i \quad \forall i, H_i(n.\text{out}) = ys_i \\ \forall eq \in n.\text{eqs}, H \vdash eq}{\vdash f(xs) \Downarrow ys}$$

## Node Semantics

$$\frac{\text{node}(G, f) = n \quad \forall i, H_i(n.\text{in}) = xs_i \quad \forall i, H_i(n.\text{out}) = ys_i \\ \forall eq \in n.\text{eqs}, H \vdash eq}{\vdash f(xs) \Downarrow ys}$$

## Node Semantics

$$\frac{\text{node}(G, f) = n \quad \forall i, H_i(n.\text{in}) = xs_i \quad \forall i, H_i(n.\text{out}) = ys_i \quad \forall eq \in n.\text{eqs}, H \vdash eq}{\vdash f(xs) \Downarrow ys}$$

## Node Semantics

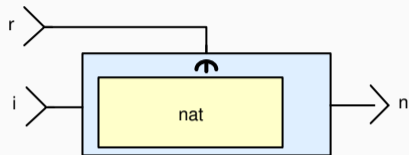
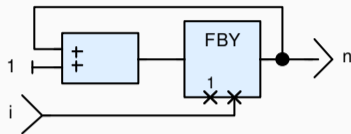
$$\frac{\text{node}(G, f) = n \quad \forall i, H_i(n.\text{in}) = xs_i \quad \forall i, H_i(n.\text{out}) = ys_i \\ \forall eq \in n.\text{eqs}, H \vdash eq}{\vdash f(xs) \Downarrow ys}$$

## Node Semantics

$$\frac{\text{node}(G, f) = n \quad \forall i, H_i(n.\text{in}) = xs_i \quad \forall i, H_i(n.\text{out}) = ys_i \\ \forall eq \in n.\text{eqs}, H \vdash eq}{\vdash f(xs) \Downarrow ys}$$

## A SIMPLER EXAMPLE: INTUITIVE SEMANTICS OF THE MODULAR RESET

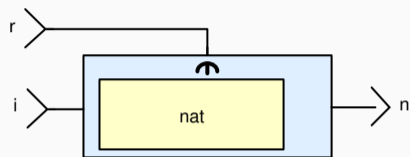
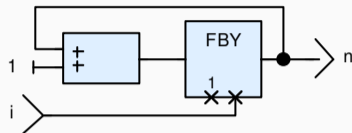
```
node nat(i: int)
  returns (n: int)
let
  n = i fby (n + 1);
tel
```





## A SIMPLER EXAMPLE: INTUITIVE SEMANTICS OF THE MODULAR RESET

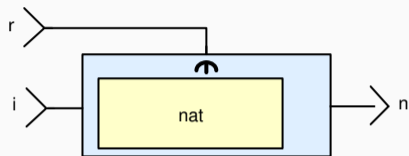
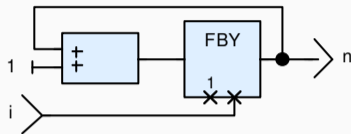
```
node nat(i: int)
  returns (n: int)
let
  n = i fby (n + 1);
tel
```



$r$	F
$i$	0
<hr/>	
$nat(i)$	0
$(\text{restart } nat \text{ every } r)(i)$	0

## A SIMPLER EXAMPLE: INTUITIVE SEMANTICS OF THE MODULAR RESET

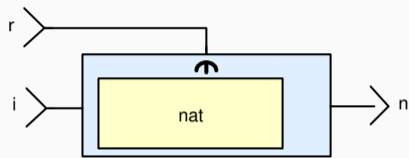
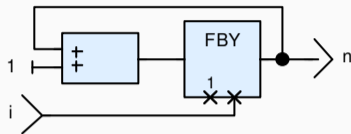
```
node nat(i: int)
  returns (n: int)
let
  n = i fby (n + 1);
tel
```



<i>r</i>	F	F
<i>i</i>	0	5
<hr/>		
<i>nat</i> ( <i>i</i> )	0	1
( <b>restart</b> <i>nat</i> <b>every</b> <i>r</i> )( <i>i</i> )	0	1

## A SIMPLER EXAMPLE: INTUITIVE SEMANTICS OF THE MODULAR RESET

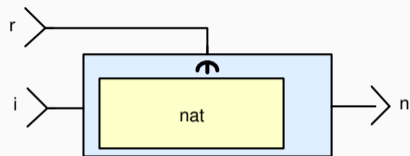
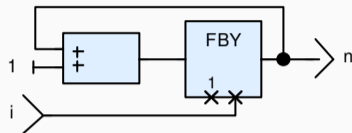
```
node nat(i: int)
  returns (n: int)
let
  n = i fby (n + 1);
tel
```



<i>r</i>	F	F	T
<i>i</i>	0	5	10
<hr/>			
<i>nat</i> ( <i>i</i> )	0	1	2
( <code>restart nat every r</code> )( <i>i</i> )	0	1	10

## A SIMPLER EXAMPLE: INTUITIVE SEMANTICS OF THE MODULAR RESET

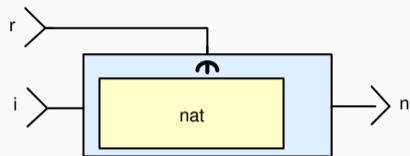
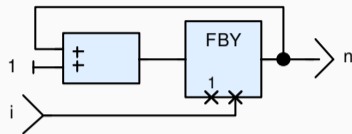
```
node nat(i: int)
  returns (n: int)
let
  n = i fby (n + 1);
tel
```



<i>r</i>	F	F	T	F
<i>i</i>	0	5	10	15
<hr/>				
<i>nat</i> ( <i>i</i> )	0	1	2	3
( <b>restart nat every r</b> )( <i>i</i> )	0	1	10	11

## A SIMPLER EXAMPLE: INTUITIVE SEMANTICS OF THE MODULAR RESET

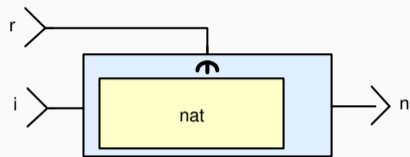
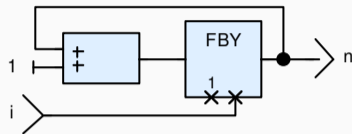
```
node nat(i: int)
  returns (n: int)
let
  n = i fby (n + 1);
tel
```



<i>r</i>	F	F	T	F	F
<i>i</i>	0	5	10	15	20
<hr/>					
<i>nat</i> ( <i>i</i> )	0	1	2	3	4
( <b>restart</b> <i>nat</i> <b>every</b> <i>r</i> )( <i>i</i> )	0	1	10	11	12

## A SIMPLER EXAMPLE: INTUITIVE SEMANTICS OF THE MODULAR RESET

```
node nat(i: int)
  returns (n: int)
let
  n = i fby (n + 1);
tel
```

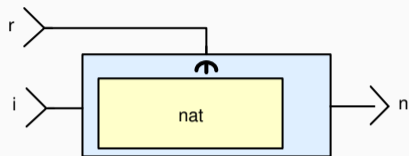
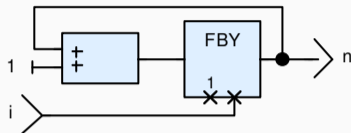


<i>r</i>	F	F	T	F	F	T
<i>i</i>	0	5	10	15	20	25
<hr/>						
<i>nat(i)</i>	0	1	2	3	4	5
<i>(restart nat every r)(i)</i>	0	1	10	11	12	25

# A SIMPLER EXAMPLE: INTUITIVE SEMANTICS OF THE MODULAR RESET

```

node nat(i: int)
  returns (n: int)
let
  n = i fby (n + 1);
tel
  
```



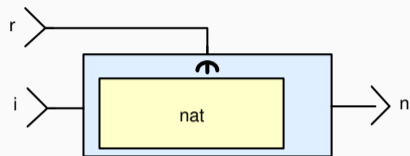
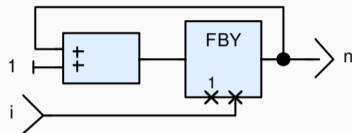
<i>r</i>	F	F	T	F	F	T	F
<i>i</i>	0	5	10	15	20	25	30
<hr/>							
<i>nat</i> ( <i>i</i> )	0	1	2	3	4	5	6
( <b>restart</b> <i>nat</i> <b>every</b> <i>r</i> )( <i>i</i> )	0	1	10	11	12	25	26

# A SIMPLER EXAMPLE: INTUITIVE SEMANTICS OF THE MODULAR RESET

```

node nat(i: int)
  returns (n: int)
let
  n = i fby (n + 1);
tel

```



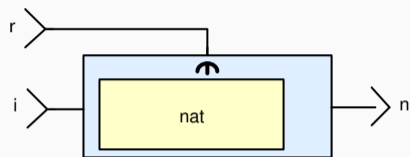
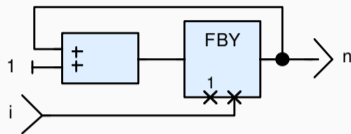
<i>r</i>	F	F	T	F	F	T	F	...
<i>i</i>	0	5	10	15	20	25	30	...
<hr/>								
<i>nat</i> ( <i>i</i> )	0	1	2	3	4	5	6	...
( <b>restart nat every <i>r</i></b> )( <i>i</i> )	0	1	10	11	12	25	26	...



```

node nat(i: int)
  returns (n: int)
let
  n = i fby (n + 1);
tel

```



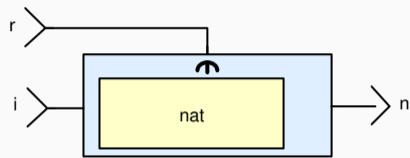
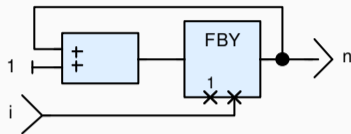
<i>r</i>	F	F	T	F	F	T	F	...
<i>i</i>	0	5	10	15	20	25	30	...
<i>nat</i> ( <i>i</i> )	0	1	2	3	4	5	6	...
( <b>restart nat every r</b> )( <i>i</i> )	0	1	10	11	12	25	26	...

Can be implemented in a higher-order recursive language

```

node nat(i: int)
  returns (n: int)
let
  n = i fby (n + 1);
tel

```



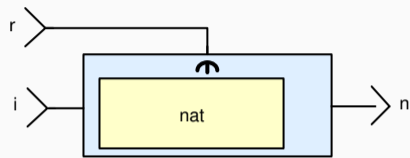
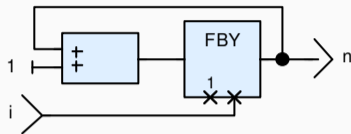
<i>r</i>	F	F	T	F	F	T	F	...
<i>i</i>	0	5	10	15	20	25	30	...
<i>nat</i> ( <i>i</i> )	0	1	2	3	4	5	6	...
( <b>restart nat every r</b> )( <i>i</i> )	0	1	10	11	12	25	26	...

Can be implemented in a higher-order recursive language

```

node nat(i: int)
  returns (n: int)
let
  n = i fby (n + 1);
tel

```



<i>r</i>	F	F	T	F	F	T	F	...
<i>i</i>	0	5	10	15	20	25	30	...
<i>nat</i> ( <i>i</i> )	0	1	2	3	4	5	6	...
( <b>restart nat every r</b> )( <i>i</i> )	0	1	10	11	12	25	26	...

Can be implemented in a higher-order recursive language

## A SIMPLER EXAMPLE: INTUITIVE SEMANTICS OF THE MODULAR RESET

$r$		F	F	T	F	F	T	F	...
<hr/>									
$i$		0	5	10	15	20	25	30	...

`(restart nat every r)(i)` 0 1 10 11 12 25 26 ...

## A SIMPLER EXAMPLE: INTUITIVE SEMANTICS OF THE MODULAR RESET

$r$	F	F	T	F	F	T	F	...
$\text{count } r$	0	0	1	1	1	2	2	...
$i$	0	5	10	15	20	25	30	...

$(\text{restart nat every } r)(i)$  0 1 10 11 12 25 26 ...

## A SIMPLER EXAMPLE: INTUITIVE SEMANTICS OF THE MODULAR RESET

$r$	F	F	T	F	F	T	F	...
$\text{count } r$	0	0	1	1	1	2	2	...
$i$	0	5	10	15	20	25	30	...
$\text{mask}_r^0 i$	0	5						...

$(\text{restart nat every } r)(i)$  0 1 10 11 12 25 26 ...

## A SIMPLER EXAMPLE: INTUITIVE SEMANTICS OF THE MODULAR RESET

$r$	F	F	T	F	F	T	F	...
$\text{count } r$	0	0	1	1	1	2	2	...
$i$	0	5	10	15	20	25	30	...
$\text{mask}_r^0 i$	0	5						...
$\text{nat}(\text{mask}_r^0 i)$	0	1						...

$(\text{restart } \text{nat every } r)(i)$  0 1 10 11 12 25 26 ...

## A SIMPLER EXAMPLE: INTUITIVE SEMANTICS OF THE MODULAR RESET

$r$	F	F	T	F	F	T	F	...
$\text{count } r$	0	0	1	1	1	2	2	...
$i$	0	5	10	15	20	25	30	...
$\text{mask}_r^0 i$	0	5						...
$\text{nat}(\text{mask}_r^0 i)$	0	1						...
$\text{mask}_r^1 i$			10	15	20			...
$(\text{restart } \text{nat } \text{every } r)(i)$	0	1	10	11	12	25	26	...



## A SIMPLER EXAMPLE: INTUITIVE SEMANTICS OF THE MODULAR RESET

$r$	F	F	T	F	F	T	F	...
$\text{count } r$	0	0	1	1	1	2	2	...
$i$	0	5	10	15	20	25	30	...
$\text{mask}_r^0 i$	0	5						...
$\text{nat}(\text{mask}_r^0 i)$	0	1						...
$\text{mask}_r^1 i$			10	15	20			...
$\text{nat}(\text{mask}_r^1 i)$			10	11	12			...
$(\text{restart } \text{nat } \text{every } r)(i)$	0	1	10	11	12	25	26	...

## A SIMPLER EXAMPLE: INTUITIVE SEMANTICS OF THE MODULAR RESET

$r$	F	F	T	F	F	T	F	...
$\text{count } r$	0	0	1	1	1	2	2	...
$i$	0	5	10	15	20	25	30	...
$\text{mask}_r^0 i$	0	5						...
$\text{nat}(\text{mask}_r^0 i)$	0	1						...
$\text{mask}_r^1 i$			10	15	20			...
$\text{nat}(\text{mask}_r^1 i)$			10	11	12			...
$\text{mask}_r^2 i$						25	30	...
$(\text{restart } \text{nat } \text{every } r)(i)$	0	1	10	11	12	25	26	...

## A SIMPLER EXAMPLE: INTUITIVE SEMANTICS OF THE MODULAR RESET

$r$	F	F	T	F	F	T	F	...
$\text{count } r$	0	0	1	1	1	2	2	...
$i$	0	5	10	15	20	25	30	...
$\text{mask}_r^0 i$	0	5						...
$\text{nat}(\text{mask}_r^0 i)$	0	1						...
$\text{mask}_r^1 i$			10	15	20			...
$\text{nat}(\text{mask}_r^1 i)$			10	11	12			...
$\text{mask}_r^2 i$						25	30	...
$\text{nat}(\text{mask}_r^2 i)$						25	26	...
$(\text{restart nat every } r)(i)$	0	1	10	11	12	25	26	...

## A SIMPLER EXAMPLE: INTUITIVE SEMANTICS OF THE MODULAR RESET

$r$	F	F	T	F	F	T	F	...
$\text{count } r$	0	0	1	1	1	2	2	...
$i$	0	5	10	15	20	25	30	...
$\text{mask}_r^0 i$	0	5						...
$\text{nat}(\text{mask}_r^0 i)$	0	1						...
$\text{mask}_r^1 i$			10	15	20			...
$\text{nat}(\text{mask}_r^1 i)$			10	11	12			...
$\text{mask}_r^2 i$						25	30	...
$\text{nat}(\text{mask}_r^2 i)$						25	26	...
$\vdots$								
$(\text{restart nat every } r)(i)$	0	1	10	11	12	25	26	...

## Node instantiation

$$\frac{\forall i, H_i \vdash \mathbf{e} \downarrow xs_i \quad \vdash f(xs) \Downarrow ys \quad \forall i, H_i(\mathbf{x}) = ys_i}{H \vdash \mathbf{x} = f(\mathbf{e})}$$

Node instantiation

$$\frac{\forall i, H_i \vdash e \downarrow xs_i \quad \vdash f(xs) \Downarrow ys \quad \forall i, H_i(x) = ys_i}{H \vdash x = f(e)}$$

Modular reset

---


$$H \vdash x = (\text{restart } f \text{ every } y)(e)$$

## Node instantiation

$$\frac{\forall i, H_i \vdash e \downarrow xs_i \quad \vdash f(xs) \Downarrow ys \quad \forall i, H_i(\mathbf{x}) = ys_i}{H \vdash \mathbf{x} = f(e)}$$

## Modular reset

$$\frac{\forall i, H_i \vdash e \downarrow xs_i \quad \forall i, H_i(\mathbf{x}) = ys_i}{H \vdash \mathbf{x} = (\text{restart } f \text{ every } y)(e)}$$

## Node instantiation

$$\frac{\forall i, H_i \vdash e \downarrow xs_i \quad \vdash f(xs) \Downarrow ys \quad \forall i, H_i(x) = ys_i}{H \vdash x = f(e)}$$

## Modular reset

$$\frac{\forall i, H_i \vdash e \downarrow xs_i \quad \forall i, H_i(y) = rs_i \quad r = \text{bools-of } rs \quad \forall k, \vdash f(\text{mask}_r^k xs) \Downarrow \text{mask}_r^k ys \quad \forall i, H_i(x) = ys_i}{H \vdash x = (\text{restart } f \text{ every } y)(e)}$$



## Node instantiation

$$\frac{\forall i, H_i \vdash e \downarrow xs_i \quad \vdash f(xs) \Downarrow ys \quad \forall i, H_i(x) = ys_i}{H \vdash x = f(e)}$$

## Modular reset

$$\frac{\forall i, H_i(y) = rs_i \quad r = \text{bools-of } rs \quad \forall i, H_i \vdash e \downarrow xs_i \quad \forall k, \vdash f(\text{mask}_r^k xs) \Downarrow \text{mask}_r^k ys \quad \forall i, H_i(x) = ys_i}{H \vdash x = (\text{restart } f \text{ every } y)(e)}$$

Universally quantified relation: unbounded number of constraints

## COMPILING THE MODULAR RESET: FROM NLUSTRE TO STC

---

## A PROBLEM WITH THE COMPILATION FROM NLUSTRE TO OBC

```
node driver(x0, y0, u, v: double, r: bool)  class driver {
  returns (x, y: double)                    instance x: ins, y: ins;
  var ax, ay: bool;
let
  x, ax = (restart ins every r)(x0, u);
  y, ay = (restart ins every r)(y0, v);
tel
                                             reset() { ins(x).reset();
                                             ins(y).reset() }
                                             step(x0, y0, u, v: double, r: bool)
                                             returns (x, y: double)
                                             var ax, ay: bool
                                             {
                                             if r { ins(x).reset() };
                                             x, ax := ins(x).step(x0, u);
                                             if r { ins(y).reset() };
                                             y, ay := ins(y).step(y0, v)
                                             }
                                             }
}
```

## A PROBLEM WITH THE COMPILATION FROM NLUSTRE TO OBC

```
node driver(x0, y0, u, v: double, r: bool)
  returns (x, y: double)
  var ax, ay: bool;
let
  x, ax = (restart ins every r)(x0, u);
  y, ay = (restart ins every r)(y0, v);
tel
```

```
class driver {
  instance x: ins, y: ins;

  reset() { ins(x).reset();
           ins(y).reset() }

  step(x0, y0, u, v: double, r: bool)
    returns (x, y: double)
    var ax, ay: bool
    {
      if r { ins(x).reset() };
      x, ax := ins(x).step(x0, u);
      if r { ins(y).reset() };
      y, ay := ins(y).step(y0, v)
    }
}
```

## A PROBLEM WITH THE COMPILATION FROM NLUSTRE TO OBC

```
node driver(x0, y0, u, v: double, r: bool)
  returns (x, y: double)
  var ax, ay: bool;
let
  x, ax = (restart ins every r)(x0, u);
  y, ay = (restart ins every r)(y0, v);
tel
```

```
class driver {
  instance x: ins, y: ins;

  reset() { ins(x).reset();
           ins(y).reset() }

  step(x0, y0, u, v: double, r: bool)
    returns (x, y: double)
    var ax, ay: bool
    {
      if r { ins(x).reset() };
      x, ax := ins(x).step(x0, u);
      if r { ins(y).reset() };
      y, ay := ins(y).step(y0, v)
    }
}
```

## A PROBLEM WITH THE COMPILATION FROM NLUSTRE TO OBC

```
node driver(x0, y0, u, v: double, r: bool)
  returns (x, y: double)
  var ax, ay: bool;
let
  x, ax = (restart ins every r)(x0, u);
  y, ay = (restart ins every r)(y0, v);
tel
```

```
class driver {
  instance x: ins, y: ins;

  reset() { ins(x).reset();
           ins(y).reset() }

  step(x0, y0, u, v: double, r: bool)
    returns (x, y: double)
    var ax, ay: bool
    {
      if r { ins(x).reset() };
      x, ax := ins(x).step(x0, u);
      if r { ins(y).reset() };
      y, ay := ins(y).step(y0, v)
    }
}
```

## A PROBLEM WITH THE COMPILATION FROM NLUSTRE TO OBC

```
node driver(x0, y0, u, v: double, r: bool)
  returns (x, y: double)
  var ax, ay: bool;
let
  x, ax = (restart ins every r)(x0, u);
  y, ay = (restart ins every r)(y0, v);
tel
```

scheduling *and* introducing state

**VS**

introducing state *then* scheduling

```
class driver {
  instance x: ins, y: ins;

  reset() { ins(x).reset();
           ins(y).reset() }

  step(x0, y0, u, v: double, r: bool)
  returns (x, y: double)
  var ax, ay: bool
  {
    if r { ins(x).reset() };
    x, ax := ins(x).step(x0, u);
    if r { ins(y).reset() };
    y, ay := ins(y).step(y0, v)
  }
}
```

Propose a new intermediate language

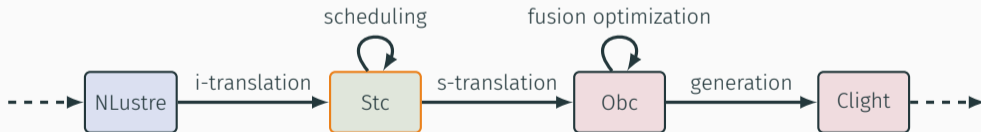
- **Invariant semantics** under permutation
- **Separate reset** construct
- **Explicit** state variables and instances



# STC: SYNCHRONOUS TRANSITION CODE

Propose a new intermediate language

- **Invariant semantics** under permutation
- **Separate reset** construct
- **Explicit** state variables and instances



```
node ins(gps, xv: double)
  returns (x: double, alarm: bool)
  var k: int, px: double,
      xe: double when not alarm;
let
  k = 0 fby k + 1;
  alarm = (k >= 50);
  xe = euler(gps when not alarm,
             xv when not alarm);
  x = merge alarm (px when alarm) xe;
  px = 0. fby x;
tel
```

```
system ins {
  init k = 0, px = 0.;
  sub xe: euler;

  transition(gps, xv: double)
    returns (x: double, alarm: bool)
    var xe: double when not alarm;
    {
      next k = k + 1;
      alarm = (k >= 50);
      xe = euler<xe>(gps when not alarm,
                    xv when not alarm);
      x = merge alarm (px when alarm) xe;
      next px = x;
    }
}
```

```

node ins(gps, xv: double)
  returns (x: double, alarm: bool)
  var k: int, px: double,
      xe: double when not alarm;
let
  k = 0 fby k + 1;
  alarm = (k >= 50);
  xe = euler(gps when not alarm,
             xv when not alarm);
  x = merge alarm (px when alarm) xe;
  px = 0. fby x;
tel

```

```

system ins {
  init k = 0, px = 0.;
  sub xe: euler;

  transition(gps, xv: double)
    returns (x: double, alarm: bool)
    var xe: double when not alarm;
    {
      next k = k + 1;
      alarm = (k >= 50);
      xe = euler<xe>(gps when not alarm,
                    xv when not alarm);
      x = merge alarm (px when alarm) xe;
      next px = x;
    }
}

```

```

node ins(gps, xv: double)
  returns (x: double, alarm: bool)
  var k: int, px: double,
      xe: double when not alarm;
let
  k = 0 fby k + 1;
  alarm = (k >= 50);
  xe = euler(gps when not alarm,
             xv when not alarm);
  x = merge alarm (px when alarm) xe;
  px = 0. fby x;
tel

```

only introducing state

```

system ins {
  init k = 0, px = 0.;
  sub xe: euler;

  transition(gps, xv: double)
    returns (x: double, alarm: bool)
    var xe: double when not alarm;
    {
      next k = k + 1;
      alarm = (k >= 50);
      xe = euler<xe>(gps when not alarm,
                    xv when not alarm);
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```

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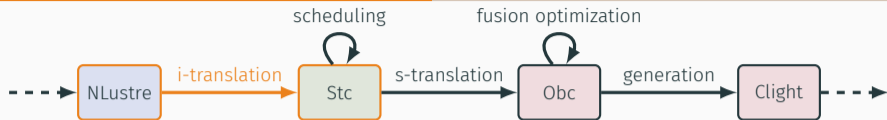
only introducing state

```

system ins {
  init k = 0, px = 0.;
  sub xe: euler;
  transition(gps, xv: double)
    returns (x: double, alarm: bool)
    var xe: double when not alarm;
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```

# COMPILATION OF THE RESET EXAMPLE



```
node driver(x0, y0, u, v: double, r: bool)
  returns (x, y: double)
  var ax, ay: bool;
let
  x, ax = (restart ins every r)(x0, u);
  y, ay = (restart ins every r)(y0, v);
tel
```

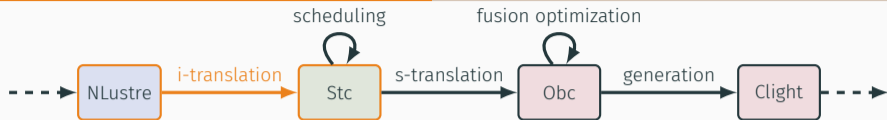
only introducing state

```
system driver {
  sub x: ins, y: ins;

  transition(x0, y0, u, v: double, r: bool)
    returns (x, y: double)
    var ax, ay: bool;
    {
      x, ax = ins<x>(x0, u);
      reset ins<x> every (. on r);
      y, ay = ins<y>(y0, v);
      reset ins<y> every (. on r);
    }
}
```



# COMPILATION OF THE RESET EXAMPLE



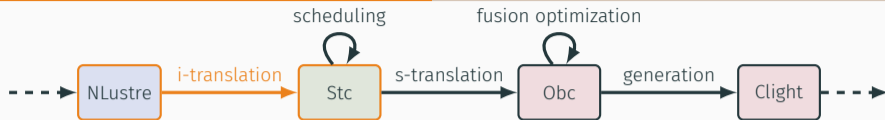
```
node driver(x0, y0, u, v: double, r: bool)
  returns (x, y: double)
  var ax, ay: bool;
let
  x, ax = (restart ins every r)(x0, u);
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```

# COMPILATION OF THE RESET EXAMPLE



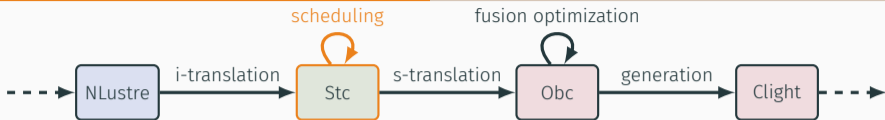
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  var ax, ay: bool;
let
  x, ax = (restart ins every r)(x0, u);
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  transition(x0, y0, u, v: double, r: bool)
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    }
}
```

# COMPILATION OF THE RESET EXAMPLE



```
node driver(x0, y0, u, v: double, r: bool)
  returns (x, y: double)
  var ax, ay: bool;
let
  x, ax = (restart ins every r)(x0, u);
  y, ay = (restart ins every r)(y0, v);
tel
```

only scheduling

```
system driver {
  sub x: ins, y: ins;

  transition(x0, y0, u, v: double, r: bool)
    returns (x, y: double)
    var ax, ay: bool;
    {
      reset ins<x> every (. on r);
      reset ins<y> every (. on r);
      x, ax = ins<x>(x0, u);
      y, ay = ins<y>(y0, v);
    }
}
```

## Transition system

- Start state  $S$ , end state  $S'$
- Transition constraints
- Transient state  $I$

## Transition system

- Start state  $S$ , end state  $S'$
- Transition constraints
- Transient state  $I$

```
system driver {  
  sub x: ins, y: ins;
```

```
  transition(x0, y0, u, v: double, r: bool)
```

```
    returns (x, y: double)
```

```
    var ax, ay: bool;
```

```
  {
```

```
    x, ax = ins<x>(x0, u);
```

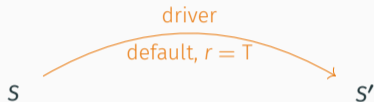
```
    reset ins<x> every (. on r);
```

```
    y, ay = ins<y>(y0, v);
```

```
    reset ins<y> every (. on r);
```

```
  }
```

```
}
```

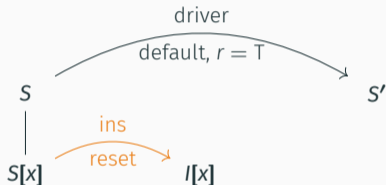


# STC INTUITIVE SEMANTICS

## Transition system

- Start state  $S$ , end state  $S'$
- Transition constraints
- Transient state  $I$

```
system driver {  
  sub x: ins, y: ins;  
  
  transition(x0, y0, u, v: double, r: bool)  
    returns (x, y: double)  
    var ax, ay: bool;  
  {  
    x, ax = ins<x>(x0, u);  
    reset ins<x> every (. on r);  
    y, ay = ins<y>(y0, v);  
    reset ins<y> every (. on r);  
  }  
}
```

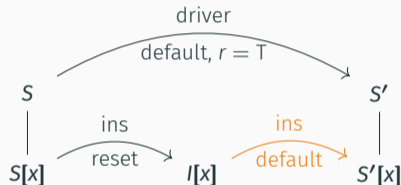


# STC INTUITIVE SEMANTICS

## Transition system

- Start state  $S$ , end state  $S'$
- Transition constraints
- Transient state  $I$

```
system driver {  
  sub x: ins, y: ins;  
  
  transition(x0, y0, u, v: double, r: bool)  
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    var ax, ay: bool;  
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    x, ax = ins<x>(x0, u);  
    reset ins<x> every (. on r);  
    y, ay = ins<y>(y0, v);  
    reset ins<y> every (. on r);  
  }  
}
```

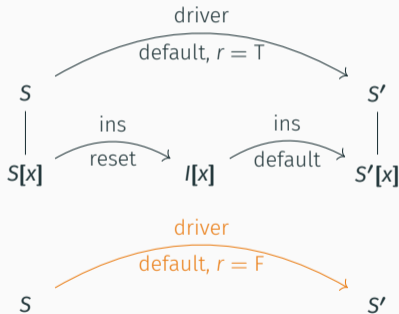


# STC INTUITIVE SEMANTICS

## Transition system

- Start state  $S$ , end state  $S'$
- Transition constraints
- Transient state  $I$

```
system driver {  
  sub x: ins, y: ins;  
  
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  }  
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```



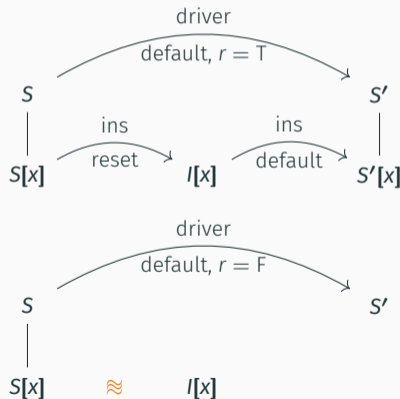


# STC INTUITIVE SEMANTICS

## Transition system

- Start state  $S$ , end state  $S'$
- Transition constraints
- Transient state  $I$

```
system driver {  
  sub x: ins, y: ins;  
  
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    reset ins<x> every (. on r);  
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  }  
}
```



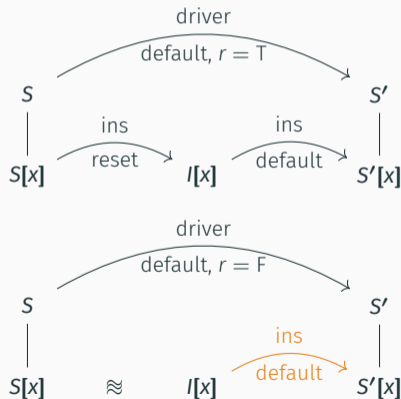
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- Start state  $S$ , end state  $S'$
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    reset ins<y> every (. on r);
  }
}
    
```



## Basic transition constraint

$$\frac{R \vdash e \downarrow R(x)}{R, S, I, S' \vdash x = e}$$

## Next transition constraint

$$\frac{R \vdash e \downarrow \langle v \rangle \quad R(x) = \langle S(x) \rangle \quad S'(x) = v}{R, S, I, S' \vdash \text{next } x = e}$$

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## Default transition

$$\frac{R \vdash e \downarrow v \quad I[i], S'[i] \vdash f(v) \Downarrow R(x) \quad \text{if } (k = 0) \text{ then } I[i] \approx S[i]}{R, S, I, S' \vdash x = f\langle i, k \rangle(e)}$$

## Reset transition

$$\frac{R \vdash ck \downarrow \text{true} \quad \text{initial-state } f\ I[i]}{R, S, I, S' \vdash \text{reset } f\langle i \rangle \text{ every } ck}$$

$$\frac{R \vdash ck \downarrow \text{false} \quad I[i] \approx S[i]}{R, S, I, S' \vdash \text{reset } f\langle i \rangle \text{ every } ck}$$

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## System

$$\frac{\text{system}(P, f) = s \quad R(\text{s.in}) = xs \quad R(\text{s.out}) = ys \\ \forall tc \in \text{s.tcs}, R, S, I, S' \vdash tc}{S, S' \vdash f(xs) \Downarrow ys}$$

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Loop

$$\frac{S, S' \vdash f(xs_n) \Downarrow ys_n \quad S' \vdash f(xs) \overset{n+1}{Q} ys}{S \vdash f(xs) \overset{n}{Q} ys}$$



Loop

$$\frac{S, S' \vdash f(xs_n) \Downarrow ys_n \quad S' \vdash f(xs) \overset{n+1}{Q} ys}{S \vdash f(xs) \overset{n}{Q} ys}$$

Loop

$$\frac{
 \frac{
 S', S'' \vdash f(xS_{n+1}) \Downarrow yS_{n+1} \quad S'' \vdash f(xs) \overset{n+2}{Q} ys
 }{
 S, S' \vdash f(xS_n) \Downarrow yS_n \quad S' \vdash f(xs) \overset{n+1}{Q} ys
 }{
 S \vdash f(xs) \overset{n}{Q} ys
 }$$

Loop

$$\begin{array}{c}
 S'', S''' \vdash f(xS_{n+2}) \Downarrow yS_{n+2} \quad S''' \vdash f(xs) \overset{n+3}{\mathbb{Q}} ys \\
 \hline
 S', S'' \vdash f(xS_{n+1}) \Downarrow yS_{n+1} \quad S'' \vdash f(xs) \overset{n+2}{\mathbb{Q}} ys \\
 \hline
 S, S' \vdash f(xS_n) \Downarrow yS_n \quad S' \vdash f(xs) \overset{n+1}{\mathbb{Q}} ys \\
 \hline
 S \vdash f(xs) \overset{n}{\mathbb{Q}} ys
 \end{array}$$

Loop

$$\begin{array}{c}
 \vdots \\
 \frac{S'', S''' \vdash f(xS_{n+2}) \Downarrow yS_{n+2} \quad \frac{S''' \vdash f(xs) \overset{n+3}{Q} ys}{\phantom{S'' \vdash f(xs) \overset{n+2}{Q} ys}}}{S', S'' \vdash f(xS_{n+1}) \Downarrow yS_{n+1} \quad S'' \vdash f(xs) \overset{n+2}{Q} ys} \\
 \frac{S, S' \vdash f(xS_n) \Downarrow yS_n \quad S' \vdash f(xs) \overset{n+1}{Q} ys}{S \vdash f(xs) \overset{n}{Q} ys}
 \end{array}$$

## CORRECTNESS: PRESERVATION OF THE SEMANTICS



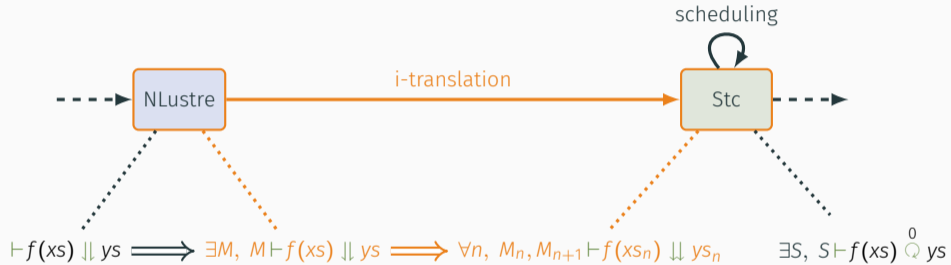
# CORRECTNESS: PRESERVATION OF THE SEMANTICS



# CORRECTNESS: PRESERVATION OF THE SEMANTICS

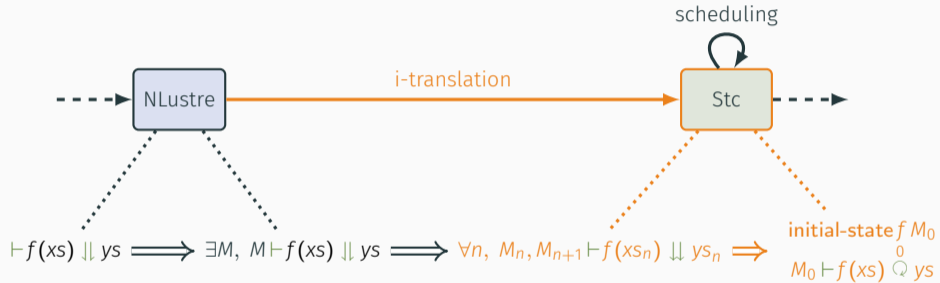


# CORRECTNESS: PRESERVATION OF THE SEMANTICS

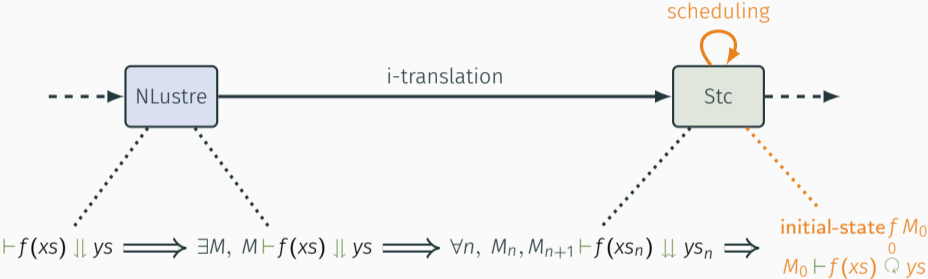




# CORRECTNESS: PRESERVATION OF THE SEMANTICS



# CORRECTNESS: PRESERVATION OF THE SEMANTICS



# PRODUCING IMPERATIVE CODE: FROM STC TO OBC

---

```

system ins {
  init k = 0, px = 0.;
  sub xe: euler;

  transition(gps, xv: double)
    returns (x: double, alarm: bool)
    var xe: double when not alarm;
  {
    alarm = (k >= 50);
    next k = k + 1;
    xe = euler<xe>(gps when not alarm,
                  xv when not alarm);
    x = merge alarm (px when alarm) xe;
    next px = x;
  }
}

```

```

class ins {
  state k: int, px: double;
  instance xe: euler;

  reset() { state(k) := 0;
           state(px) := 0.;
           euler(xe).reset() }

  step(gps, xv: double)
    returns (x: double, alarm: bool)
    var xe: double
  {
    alarm := state(k) >= 50;
    state(k) := state(k) + 1;
    if alarm { }
    else { xe := euler(xe).step(gps, xv) };
    if alarm { x := state(px) }
    else { x := xe };
    state(px) := x
  }
}

```

```

system ins {
  init k = 0, px = 0.;
  sub xe: euler;

  transition(gps, xv: double)
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    else { xe := euler(xe).step(gps, xv) };
    if alarm { x := state(px) }
    else { x := xe };
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```

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    if alarm { }
    else { xe := euler(xe).step(gps, xv) };
    if alarm { x := state(px) }
    else { x := xe };
    state(px) := x
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}

```



```

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  init k = 0, px = 0.;
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    var xe: double when not alarm;
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    alarm = (k >= 50);
    next k = k + 1;
    xe = euler<xe>(gps when not alarm,
                  xv when not alarm);
    x = merge alarm (px when alarm) xe;
    next px = x;
  }
}

```

```

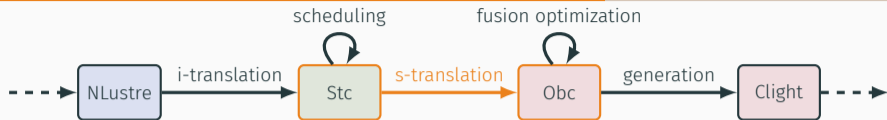
class ins {
  state k: int, px: double;
  instance xe: euler;

  reset() { state(k) := 0;
           state(px) := 0.;
           euler(xe).reset() }

  step(gps, xv: double)
    returns (x: double, alarm: bool)
    var xe: double
  {
    alarm := state(k) >= 50;
    state(k) := state(k) + 1;
    if alarm { }
    else { xe := euler(xe).step(gps, xv) };
    if alarm { x := state(px) }
    else { x := xe };
    state(px) := x
  }
}

```

# COMPILATION OF THE RESET EXAMPLE



```
system driver {
  sub x: ins, y: ins;

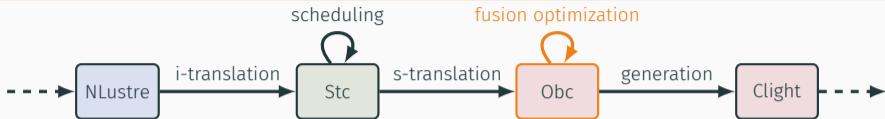
  transition(x0, y0, u, v: double, r: bool)
    returns (x, y: double)
    var ax, ay: bool;
  {
    reset ins<x> every (. on r);
    reset ins<y> every (. on r);
    x, ax = ins<x>(x0, u);
    y, ay = ins<y>(y0, v);
  }
}
```

```
class driver {
  instance x: ins, y: ins;

  reset() { ins(x).reset();
           ins(y).reset() }

  step(x0, y0, u, v: double, r: bool)
    returns (x, y: double)
    var ax, ay: bool
  {
    if r { ins(x).reset() };
    if r { ins(y).reset() };
    x, ax := ins(x).step(x0, u);
    y, ay := ins(y).step(y0, v)
  }
}
```

# COMPILATION OF THE RESET EXAMPLE



```
system driver {  
  sub x: ins, y: ins;  
  
  transition(x0, y0, u, v: double, r: bool)  
    returns (x, y: double)  
    var ax, ay: bool;  
  {  
    reset ins<x> every (. on r);  
    reset ins<y> every (. on r);  
    x, ax = ins<x>(x0, u);  
    y, ay = ins<y>(y0, v);  
  }  
}
```

```
class driver {  
  instance x: ins, y: ins;  
  
  reset() { ins(x).reset();  
           ins(y).reset() }  
  
  step(x0, y0, u, v: double, r: bool)  
    returns (x, y: double)  
    var ax, ay: bool  
  {  
    if r { ins(x).reset();  
          ins(y).reset() };  
    x, ax := ins(x).step(x0, u);  
    y, ay := ins(y).step(y0, v)  
  }  
}
```

## Expressions

$$\frac{}{me, ve \vdash x \Downarrow ve(x)} \quad \frac{}{me, ve \vdash \text{state}(x) \Downarrow me(x)} \quad \frac{me, ve \vdash e_1 \Downarrow v_1 \quad me, ve \vdash e_2 \Downarrow v_2}{me, ve \vdash e_1 + e_2 \Downarrow [[+]](v_1, v_2)}$$

## Statements

$$\frac{me, ve \vdash e \Downarrow v}{me, ve \vdash x := e \Downarrow (me, ve\{x \mapsto v\})} \quad \frac{me, ve \vdash e \Downarrow v}{me, ve \vdash \text{state}(x) := e \Downarrow (me\{x \mapsto v\}, ve)}$$

$$\frac{me, ve \vdash s_1 \Downarrow (me_1, ve_1) \quad me_1, ve_1 \vdash s_2 \Downarrow (me_2, ve_2)}{me, ve \vdash s_1 ; s_2 \Downarrow (me_2, ve_2)} \quad \frac{me, ve \vdash e \Downarrow v \quad me[i] \vdash c.f(v) \Downarrow^w me'_i}{me, ve \vdash x := c(i).f(e) \Downarrow (me\{i \mapsto me'_i\}, ve\{x \mapsto w\})}$$

## Expressions

$$\frac{}{me, ve \vdash x \Downarrow ve(x)} \quad \frac{}{me, ve \vdash \text{state}(x) \Downarrow me(x)} \quad \frac{me, ve \vdash e_1 \Downarrow v_1 \quad me, ve \vdash e_2 \Downarrow v_2}{me, ve \vdash e_1 + e_2 \Downarrow [[+]](v_1, v_2)}$$

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$$\frac{me, ve \vdash s_1 \Downarrow (me_1, ve_1) \quad me_1, ve_1 \vdash s_2 \Downarrow (me_2, ve_2)}{me, ve \vdash s_1 ; s_2 \Downarrow (me_2, ve_2)} \quad \frac{me, ve \vdash e \Downarrow v \quad me[i] \vdash c.f(v) \Downarrow^w me'_i}{me, ve \vdash x := c(i).f(e) \Downarrow (me\{i \mapsto me'_i\}, ve\{x \mapsto w\})}$$

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$$\frac{me, ve \vdash s_1 \Downarrow (me_1, ve_1) \quad me_1, ve_1 \vdash s_2 \Downarrow (me_2, ve_2)}{me, ve \vdash s_1 ; s_2 \Downarrow (me_2, ve_2)} \quad \frac{me, ve \vdash e \Downarrow v \quad me[i] \vdash c.f(v) \Downarrow^w me'_i}{me, ve \vdash x := c(i).f(e) \Downarrow (me\{i \mapsto me'_i\}, ve\{x \mapsto w\})}$$

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## Expressions

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## Statements

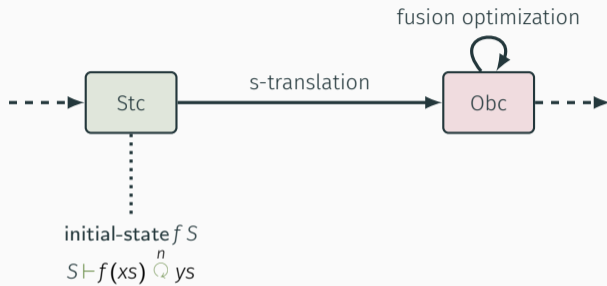
$$\frac{me, ve \vdash e \Downarrow v}{me, ve \vdash x := e \Downarrow (me, ve\{x \mapsto v\})} \quad \frac{me, ve \vdash e \Downarrow v}{me, ve \vdash \text{state}(x) := e \Downarrow (me\{x \mapsto v\}, ve)}$$

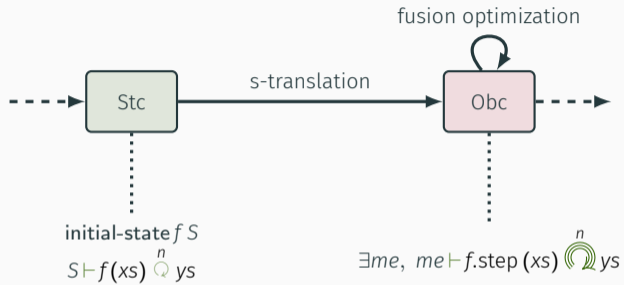
$$\frac{me, ve \vdash s_1 \Downarrow (me_1, ve_1) \quad me_1, ve_1 \vdash s_2 \Downarrow (me_2, ve_2)}{me, ve \vdash s_1 ; s_2 \Downarrow (me_2, ve_2)} \quad \frac{me, ve \vdash e \Downarrow v \quad me[i] \vdash c.f(v) \Downarrow^w me'_i}{me, ve \vdash x := c(i).f(e) \Downarrow (me\{i \mapsto me'_i\}, ve\{x \mapsto w\})}$$

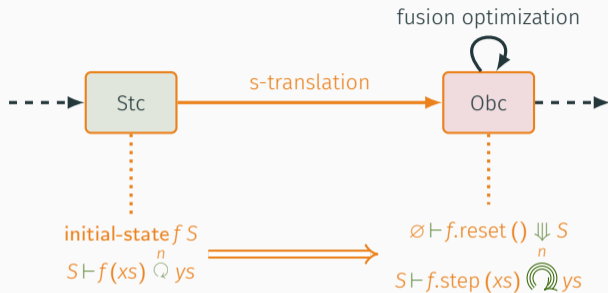
Loop

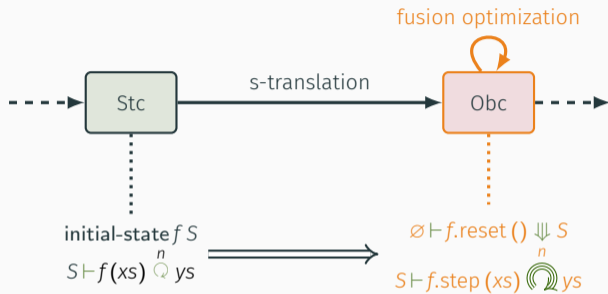
$$\frac{me \vdash c.f(xs_n) \Downarrow^{ys_n} me' \quad me' \vdash c.f(xs) \Downarrow^{n+1} ys}{me \vdash c.f(xs) \Downarrow^n ys}$$











## GENERATING CLIGHT CODE

---

## CompCert

Mechanization in Coq of the syntax, the semantics and the compilation algorithms of the C language.

## Clight

- CompCert intermediate language
- very similar to C
- low-level operations (addresses, structures,...)

```
class ins {
  state k: int, px: double;
  instance xe: euler;

  reset() { state(k) := 0;
           state(px) := 0.;
           euler(xe).reset() }

  step(gps, xv: double)
    returns (x: double, alarm: bool)
    var xe: double
  {
    alarm := state(k) >= 50;
    state(k) := state(k) + 1;
    if alarm { x := state(px) }
    else {
      xe := euler(xe).step(gps, xv);
      x := xe };
    state(px) := x
  }
}
```

```
struct ins {
  int k;
  double px;
  struct euler xe;
};

void fun$ins$reset(struct ins *self) {
  self->k = 0;
  self->px = 0;
  fun$euler$reset(&(self->xe));
  return;
}
```

```

class ins {
  state k: int, px: double;
  instance xe: euler;

  reset() { state(k) := 0;
           state(px) := 0.;
           euler(xe).reset() }

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      x := xe };
    state(px) := x
  }
}

```

```

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  int k;
  double px;
  struct euler xe;
};

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  fun$euler$reset(&(self->xe));
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}

```



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    if alarm { x := state(px) }
    else {
      xe := euler(xe).step(gps, xv);
      x := xe };
    state(px) := x
  }
}

```

```

struct fun$ins$step {
  double x;
  bool alarm;
};

void fun$ins$step(struct ins *self,
                  struct fun$ins$step *out,
                  double gps, double xv) {
  register double step$x;
  register double xe;
  out->alarm = self->k >= 50;
  self->k = self->k + 1;
  if (out->alarm) { out->x = self->px; }
  else {
    step$x = fun$euler$step(&(self->xe), gps, xv);
    xe = step$x;
    out->x = xe;
  }
  self->px = out->x;
  return;
}

```

```

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  state k: int, px: double;
  instance xe: euler;

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  out->alarm = self->k >= 50;
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    step$x = fun$euler$step(&(self->xe), gps, xv);
    xe = step$x;
    out->x = xe;
  }
  self->px = out->x;
  return;
}

```

## MAIN LOOP

```
struct nav {
    bool c;
    bool r;
    struct ins insr;
};

struct fun$nav$step {
    double x;
    bool alarm;
};

struct nav self$;
double volatile gps$;
double volatile xv$;
bool volatile s$;
double volatile x$;
bool volatile alarm$;
```

```
int main(void) {
    struct fun$nav$step out$step;
    register double gps;
    register double xv;
    register bool s;

    fun$nav$reset(&self$);

    while (true) {
        gps = volatile_load(&gps$);
        xv = volatile_load(&xv$);
        s = volatile_load(&s$);

        fun$nav$step(&self$, &out$step, gps, xv, s);

        volatile_store(&x$, out$step.x);
        volatile_store(&alarm$, out$step.alarm);
    }
}
```

# MAIN LOOP

```
struct nav {  
    bool c;  
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}
```

- contiguous blocks memory model
- variables and registers
- semantic state  $(E, L, M)$

$E$  variable environment: maps identifiers to locations

$L$  register environment: maps identifiers to values

$M$  memory: maps locations to bytes

### Consequences of CompCert's memory model

- aliasing
- alignment
- permissions
- type sizes

### Manipulation of structures and pointers

## Consequences of CompCert's memory model

- aliasing
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## Manipulation of structures and pointers

**Solution:** use Separation Logic assertions



# SEPARATION LOGIC

An extension of Hoare logic to reason about programs that manipulate pointers and structures.

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The predicate  $M \models P * Q$  asserts that  $M$  can be partitioned into two distinct areas on which  $P$  and  $Q$  hold respectively.

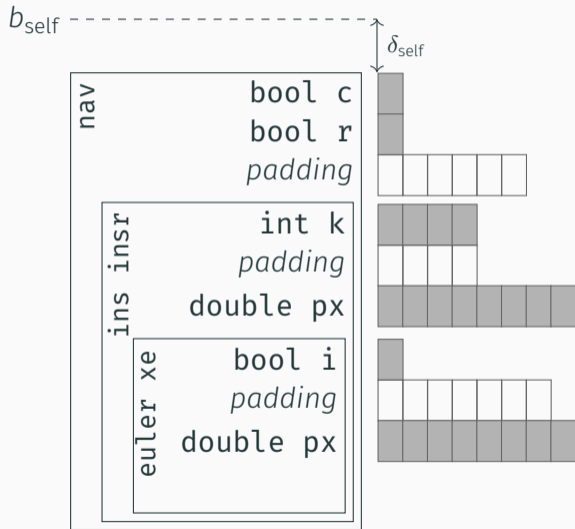
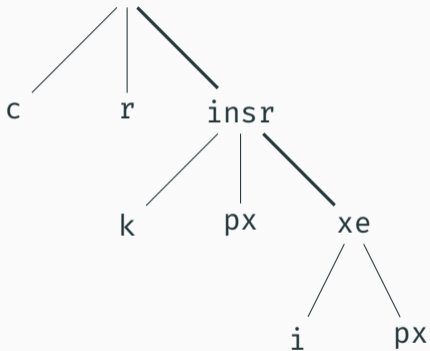
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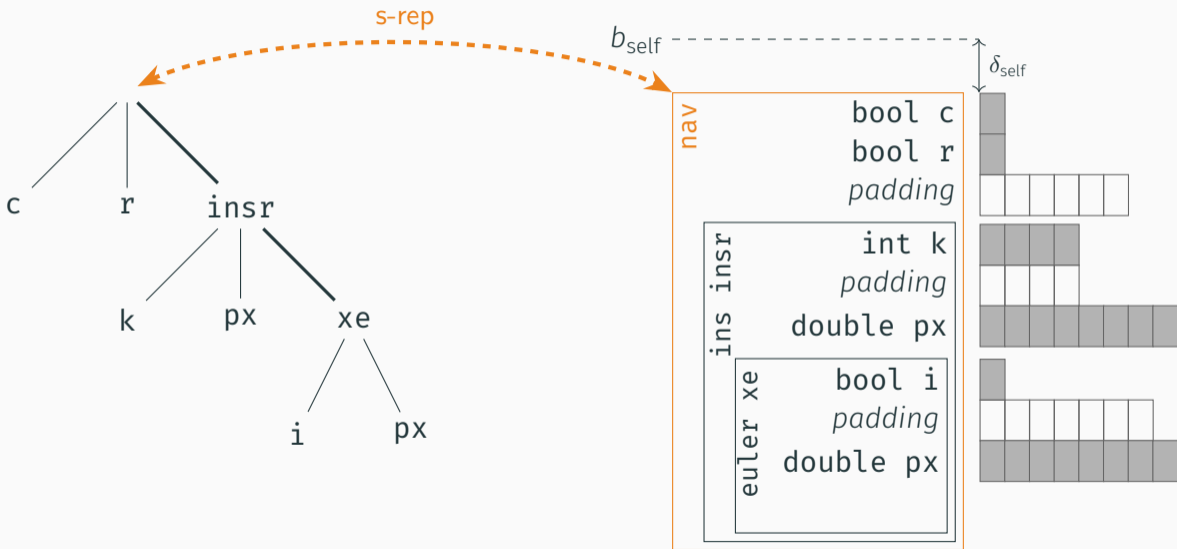
The predicate  $M \models P * Q$  asserts that  $M$  can be partitioned into two distinct areas on which  $P$  and  $Q$  hold respectively.

CompCert already uses a small Separation Logic library for one of its passes.

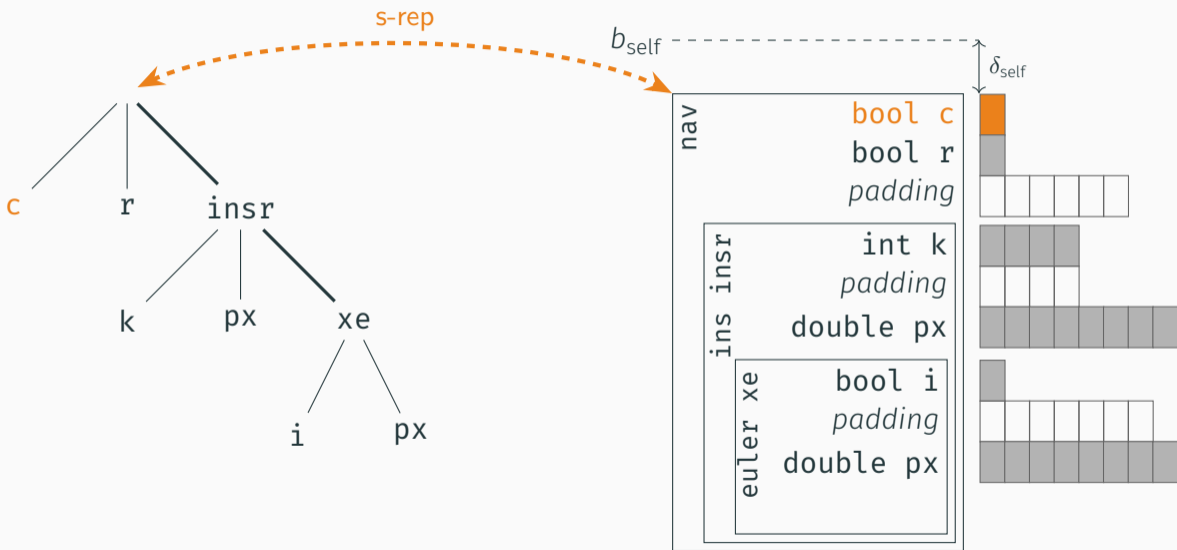
# STATE CORRESPONDENCE PREDICATE



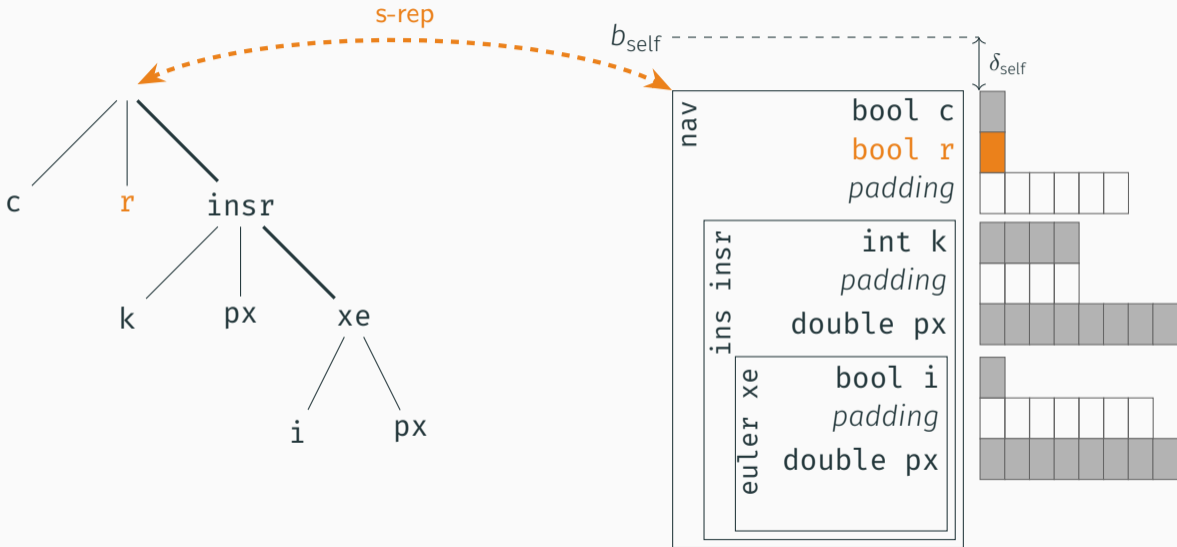
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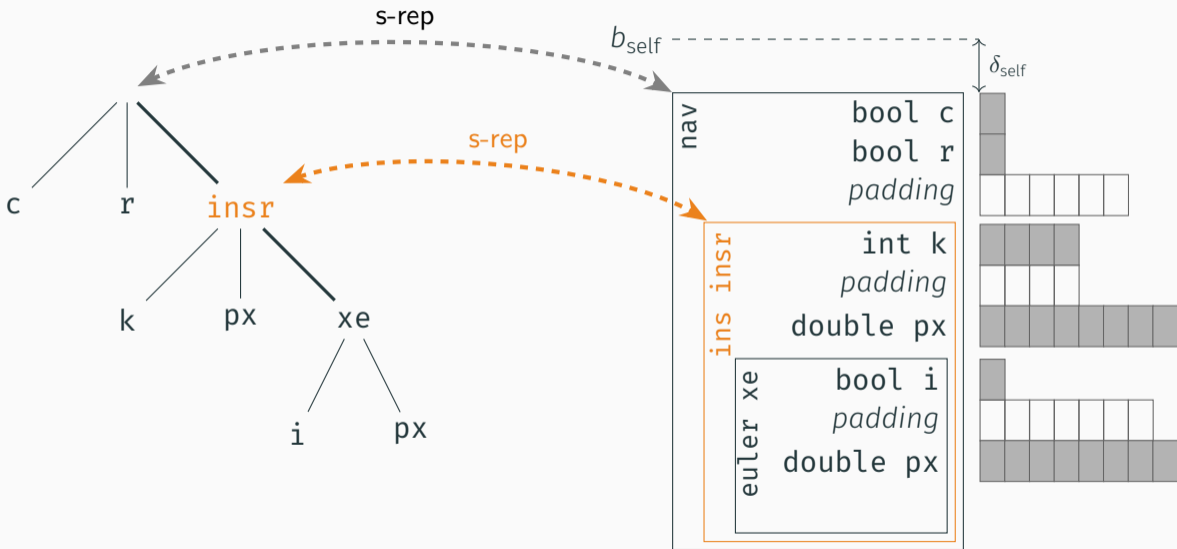
# STATE CORRESPONDENCE PREDICATE



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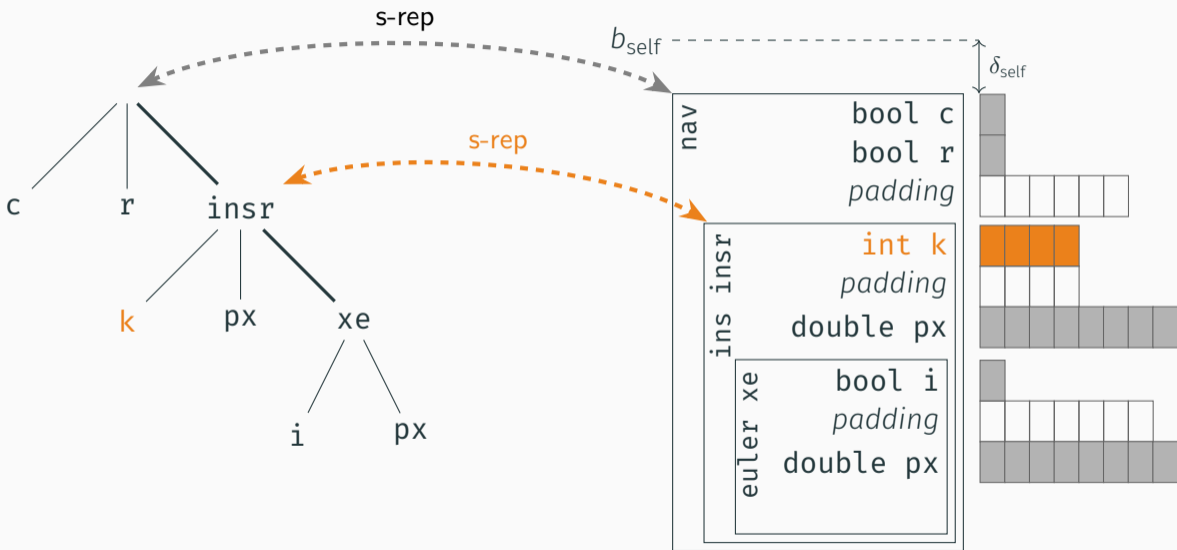


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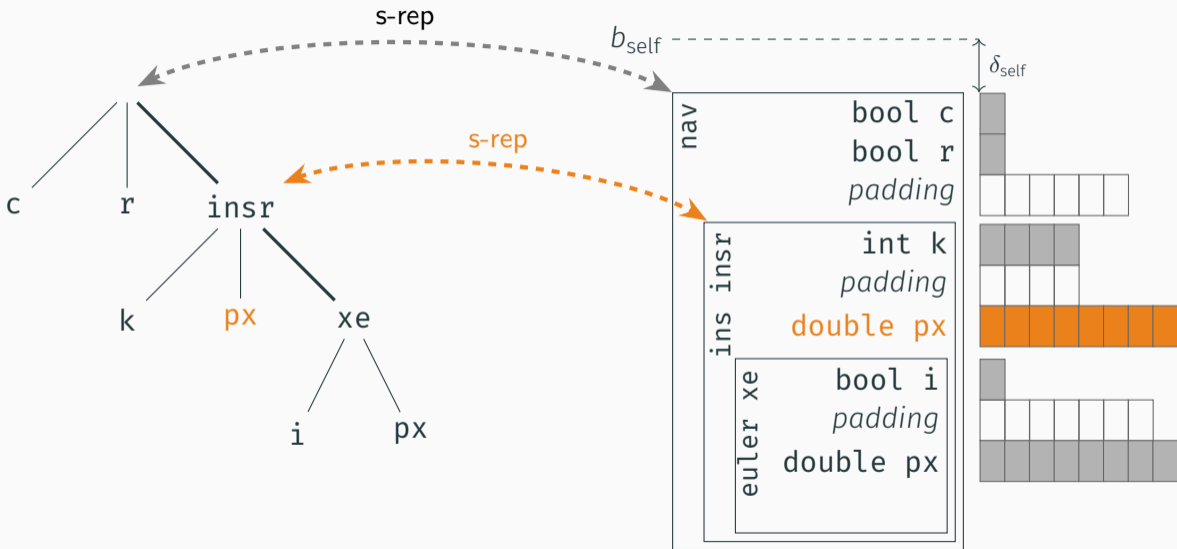




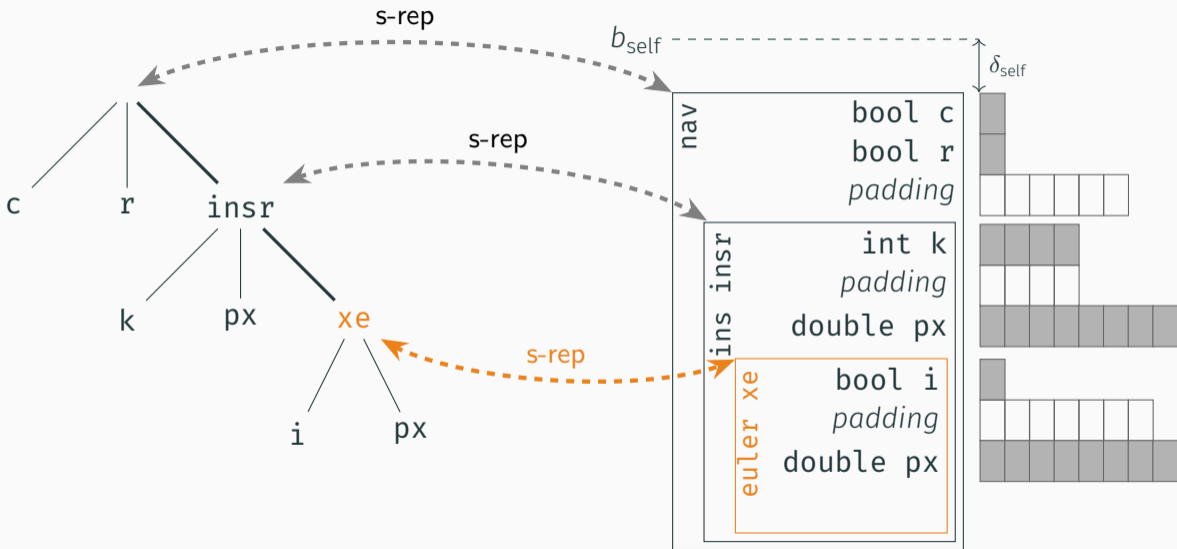
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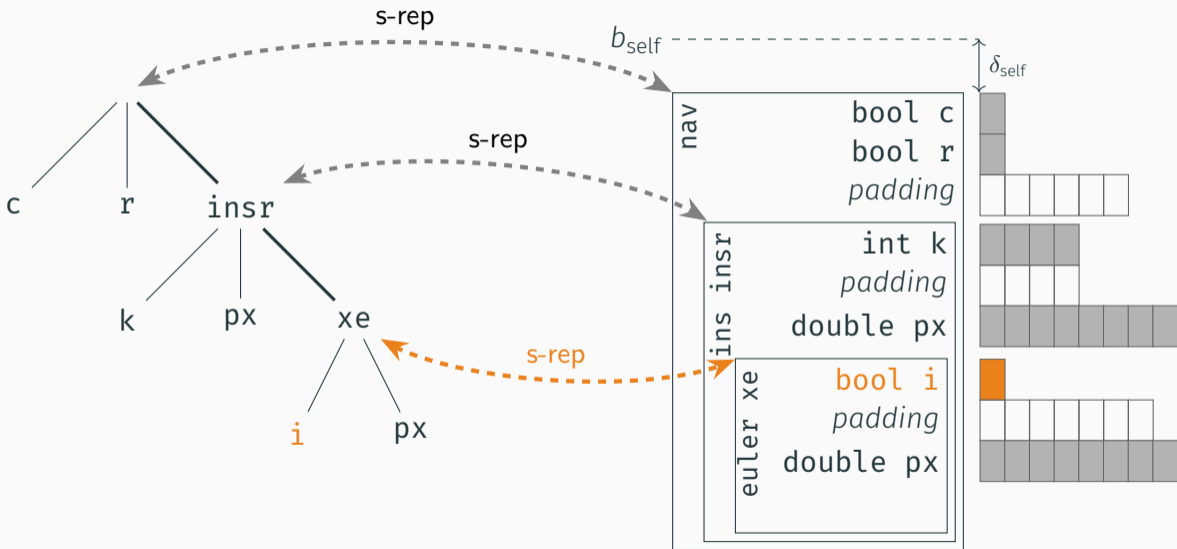
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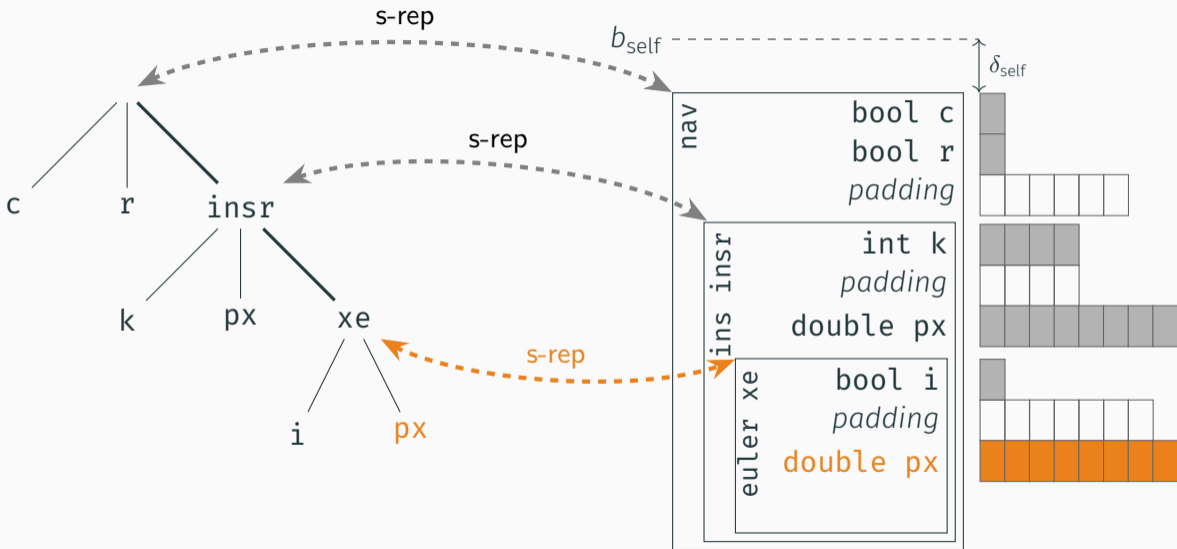
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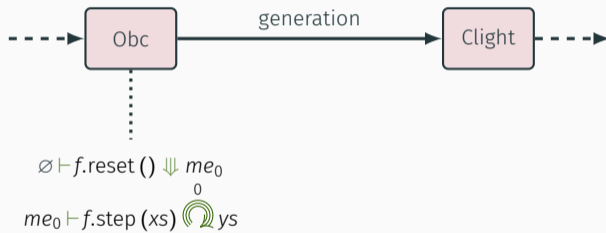


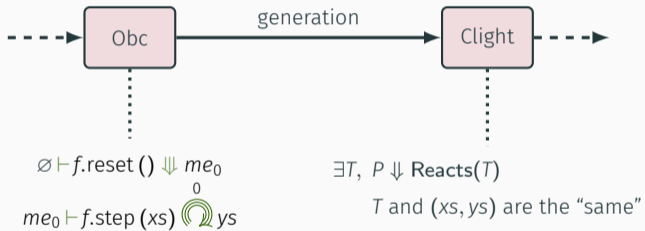
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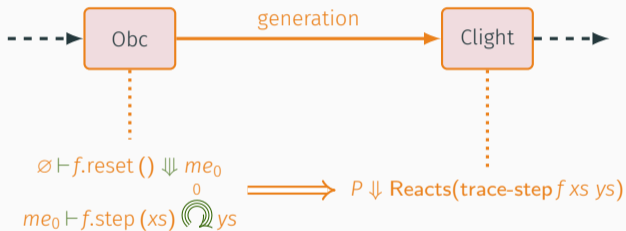


# STATE CORRESPONDENCE PREDICATE











## CONCLUSION

---

## Theorem (Vélus correctness)

Given a list of declarations  $D$ , a name  $f$ , lists of streams of values  $\mathbf{xs}$  and  $\mathbf{ys}$ , an NLustre program  $G$  and an assembly program  $P$  such that  $\text{compile } D \ f = \text{OK } (G, P)$  and  $G \vdash f(\mathbf{xs}) \Downarrow \mathbf{ys}$ , then there exists an infinite trace of events  $T$  such that

$$P \Downarrow_{\text{ASM}} \text{Reacts}(T) \quad \text{and} \quad \text{bisim-IO}^G f \ \mathbf{xs} \ \mathbf{ys} \ T$$

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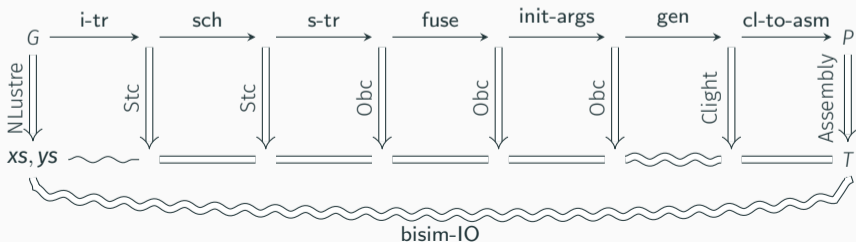
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## Theorem (Vélus correctness)

Given a list of declarations  $D$ , a name  $f$ , lists of streams of values  $\mathbf{xs}$  and  $\mathbf{ys}$ , an *NLustre* program  $G$  and an assembly program  $P$  such that  $\text{compile } D \ f = \text{OK } (G, P)$  and  $G \vdash f(\mathbf{xs}) \Downarrow \mathbf{ys}$ , then there exists an infinite trace of events  $T$  such that

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PLDI'17

## A Formally Verified Compiler for Lustre

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Xavier Leroy<sup>1</sup> Marc Pouzet<sup>1,2,1</sup> Lionel Rieg<sup>5,6</sup>  
 Xavier.Leroy@inria.fr Marc.Pouzet@ens.fr Lionel.Rieg@yale.edu

<sup>1</sup> Inria, Paris, France

<sup>2</sup> Département d'Informatique, École normale supérieure, PSL Research University, Paris, France

<sup>3</sup> Sorbonne Universités, UPMC Univ Paris 06, France

<sup>4</sup> CNRS, LIP6 UMR 7606, Paris, France

<sup>5</sup> Collège de France, Paris, France

<sup>6</sup> Yale University, New Haven, Connecticut, USA

### Abstract

The correct compilation of block diagram languages like Lustre, SCADE, and a discrete subset of Simulink is important since they are used to program critical embedded control software. We describe the specification and verification in an Interactive Theorem Prover of a compilation chain that treats the key aspects of Lustre: sampling, modes, and delays. Building on CompCert, we show that repeated execution of the generated assembly code faithfully implements the dataflow semantics of source programs.

We resolve two key technical challenges. The first is the change from a synchronous dataflow semantics, where programs manipulate streams of values, to an imperative one, where computations manipulate memory sequentially. The second is the verified compilation of an imperative language with encapsulated state to C code where the state is realized by nested records. We also treat a standard control optimization that abstracts unnecessary conditional statements.

CCS Concepts: • Software and its engineering → Design

### 1. Introduction

Lustre was introduced in 1987 as a programming language for embedded control and signal processing systems [13]. It gave rise to the industrial tool SCADE Suite<sup>1</sup> and can serve as a target to compile a subset of Simulink/Stateflow<sup>2</sup> to executable code [15, 61]. SCADE Suite is used to develop safety-critical applications like fly-by-wire controllers and power plant monitoring software. Several properties make Lustre-like languages suitable for such tasks: constructs for programming reactive controllers, execution in statically-bounded time and memory, a mathematically well-defined semantics based on dataflow streams [13], traceable and modular compilation schemes [8], and the practicability of automatic program verification [17, 25, 30, 38] and industrial certification. These languages allow engineers to develop and validate systems at the level of abstract block diagrams that are compiled directly to executable code.

Compilation transforms sets of equations that define streams of values into sequences of imperative instructions that manipulate the memory of a machine. Repeatedly exe-

SCOPES'18

## Towards a verified Lustre compiler with modular reset

Extended Abstract

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### ABSTRACT

This paper presents ongoing work to add a modular reset construct to a verified Lustre compiler. We present a novel formal specification for the construct and sketch our plans to integrate it into the compiler and its correctness proof.

### CCS CONCEPTS

• Software and its engineering → Semantics; Formal software verification; Compilers.

### KEYWORDS

Synchronous Languages (Lustre), Verified Compilation

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### 1 INTRODUCTION

Lustre is a programming language for embedded control and signal processing systems [14]. Synchronous languages like Lustre allow engineers to design and validate systems at the level of abstract block diagrams and to automatically generate executable code.

Compilation transforms sets of equations defining streams of values into imperative code. We are developing a formally verified Lustre compiler called *Méru* [3] as the Coq [6] interactive theorem prover. It integrates the CompCert C compiler [2, 7] and formally guarantees that repeated executions of the generated assembly code correctly realize the concrete values of the dataflow streams.

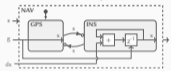


Figure 2: A graphical representation of a state machine for a simple navigation system

### 2 LUSTRE AND ITS VERIFIED COMPILER

The example in Figure 1 shows the logic of a simple navigation system, such as could be specified, for instance, in graphical tools like SCADE Suite<sup>3</sup> or Simulink.<sup>4</sup> The system takes three inputs: a data from a GPS unit,  $ds$ , a local odometric estimate, and  $is$ , a boolean input that triggers mode changes. It produces an output  $is$  giving the current position. The system has two modes: GPS uses the external data directly and INS (Inertial Navigation System) is a fallback mode where the position is estimated by adding successive  $ds$  values to the external value at mode entry.

The state machine shown in the figure can be compiled into a purely dataflow program that uses a modular reset [5]. To show why the modular reset is necessary, we start by reprogramming the example in Lustre without it:

```

mode NAV, ds, is, is', is'' : bool;
mode' : bool;
is'' = if (is) then (ds) + is' else is;
is' = if (is) then (ds) + is else is;
is = if (is) then (ds) + is else is;

```

POPL'20

## Mechanized Semantics and Verified Compilation for a Dataflow Synchronous Language with Reset

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Specifications based on block diagrams and state machines are used to design control software, especially in the certified development of safety-critical applications. Tools like SCADE Suite and Simulink/Stateflow are equipped with compilers that translate such specifications into executable code. They provide programming languages for composing functions over streams as typified by Dataflow Synchronous Languages like Lustre.

Recent work builds on CompCert to specify and verify a compiler for the core of Lustre in the Coq Interactive Theorem Prover. It formally links the stream-based semantics of the source language to the sequential memory manipulations of generated assembly code. We extend this work to treat a primitive for resetting subsystems. Our contributions include new semantic rules that are suitable for mechanized reasoning, a novel intermediate language for generating optimized code, and proofs of correctness for the associated compilation passes.

CCS Concepts: • Software and its engineering → Formal language definitions; Software verification; Compilers; • Computer systems organization → Embedded software.

Additional Key Words and Phrases: stream languages, verified compilation, interactive theorem proving

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### 1 INTRODUCTION

Block-diagram tools like SCADE Suite<sup>1</sup> and Simulink<sup>2</sup> are used to design control software. At their core are dataflow languages: operators apply point-wise to streams, state is encoded by unit delays, and subsystems are abstracted as stream functions. The Lustre synchronous language [Cas et al. 1987] epitomizes these ideas, but more sophisticated applications require more sophisticated constructs like state machines. State machines can be compiled into primitive constructs [Colgan

1<sup>st</sup> version of Vélus  
 Obsc to Clight pass

Reset formal semantics

2<sup>nd</sup> version of Vélus  
 Stc and reset compilation



## Summary

- A verified compiler from Lustre to Assembly
- A single additional semantic rule for the reset
- An intermediate transition system language: Stc



[velus.inria.fr](http://velus.inria.fr)

[github.com/INRIA/velus](https://github.com/INRIA/velus)

## Future Work

- Normalization
- State machines
- *Refinement*
- Optimizations

## Perspectives and discussion

- 42 000 loc and only 3% of functional code
- Extensibility
- Maintenance
- Axioms

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